CBSE CLASS IX · MATHEMATICS · 2026–27

MATHS IN MOTION

A Hands-On Activity Book for Class IX

37 investigations · build it, measure it, then prove it

- ✓ **Fully mapped to NCF-SE 2023 Curricular Goals and Competencies**
- ✓ **Every activity starts from a real decision someone has to make**
- ✓ **Constructions, models and manipulatives before formulae**
- ✓ **Indian Knowledge Systems woven in throughout, not appended**

Name: _____

Class: IX Section: _____ Roll No.: _____

School: _____

Session: 2026–27 Team members: _____

How to Use This Book

Most mathematics books give you the rule and then the exercises. This book reverses that. You will cut, fold, measure, roll, pour, survey and construct first — and the rule will arrive as the thing that explains what you already saw. Several activities explicitly refuse to give you the formula until you have derived it.

Every activity begins with a decision somebody actually has to make: which mobile plan, which water tank, which route, which seed, how to seat sixty people fairly. Mathematics appears as the way to settle the question, which is what it has always been for.

Proof is treated seriously here. Measuring twenty times is not proof, and this book will keep saying so. You will be asked repeatedly to distinguish between something being true in your six cases and something being true always.

Where a result was first obtained in India — the Sulbasutra constructions, Brahmagupta's formula, Virahanka's sequence, Madhava's series, Aryabhata's chords — it appears inside the activity as mathematics, not as a decorative footnote.

The seven parts of every activity

- The Situation — the real decision, with its constraints and its stakes.
- Get Your Hands On It — the physical materials. Almost everything is paper, string, card or waste.
- Explore and Build — the construction or investigation, step by step.
- Record and Compute — the working table, filled in as you go.
- Reason It Out — questions that push from 'it worked' to 'it must work'.
- Push Further — a harder case, an optimisation, or a proof to complete.
- Roots and Reach — the historical source of the idea and where it reaches outside mathematics.

Six working rules for this book

1. Estimate before you compute. Writing down a wrong estimate is more useful than writing down nothing.
2. Measure at least three times and use the mean. A single reading is an anecdote.
3. Six examples are not a proof. Say clearly which of your claims are checked and which are proved.
4. One counterexample settles a false claim. Look for one before you start proving.
5. State your units and your scale every single time. Most errors in this book will be unit errors.
6. Report the result you got, including the one that broke your model. Those are the interesting ones.

Contents

Update this field in Word (Ctrl+A, then F9) to generate page numbers.

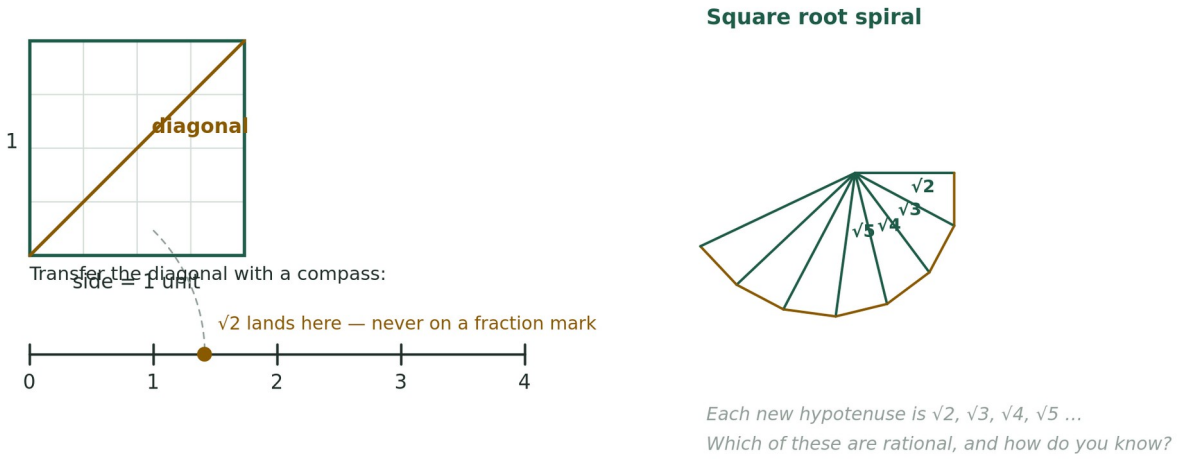
Number System

Activity 1 · The Tailor Who Cannot Measure Exactly

Number System

Competencies: C-1.1, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 1 · The unit square, its diagonal, and the square root spiral



Measure the diagonal, then try to write it as a fraction. Record every attempt and its error.

Figure 1 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A tailor's measuring tape is marked in eighths of an inch. A customer insists her sleeve must be exactly the length of the diagonal of a 1-inch square. The tailor says he can get as close as she likes but never exactly. Your team must decide whether he is making an excuse or stating a mathematical fact — and prove it.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Squared paper, ruler, compass, string

Calculator, scissors

A long strip of paper for a number line

EXPLORE AND BUILD

1. Draw a square of side 1 unit on squared paper. Draw its diagonal and measure it as accurately as your ruler permits. Record your reading.

2. Using compass and straight edge, transfer that diagonal length onto a number line starting at 0. Mark the point.
3. Now try to express your measured value as a fraction of two whole numbers. Try 7/5, then 17/12, then 41/29. Record the decimal value of each and the error against your measurement.
4. Notice that the errors shrink but never vanish. Tabulate the shrinking error.
5. Construct the square root spiral: start with a unit right triangle, then build successive right triangles each using the previous hypotenuse as a leg. Mark the lengths obtained.
6. Write out the proof by contradiction that the diagonal cannot be written as a ratio of two whole numbers with no common factor.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Fraction tried	Decimal value	Square of the value	Error from 2	Better or worse?

REASON IT OUT

Q1. At which step of the contradiction proof does the assumption actually break? Point to the exact line.

Q2. The tailor can hit any accuracy the customer names. Explain why that is not the same as being exact.

Q3. Between 1.41 and 1.42 there are infinitely many rational numbers. Write three of them, and explain the method you used.

Q4. Your square root spiral produced lengths for several roots. Which of them are rational, and how do you know?

PUSH FURTHER

Prove the irrationality of the square root of 3 by adapting your argument. Then attempt the same argument for the square root of 4 and identify precisely where it fails — that failure is the most instructive part of the exercise.

ROOTS AND REACH

The Sulbasutras give a remarkably accurate value for the diagonal of a square, expressed as a sum of unit fractions. Compute that value, compare it with your calculator, and find the error. Consider what it means to reach that accuracy without decimal notation.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

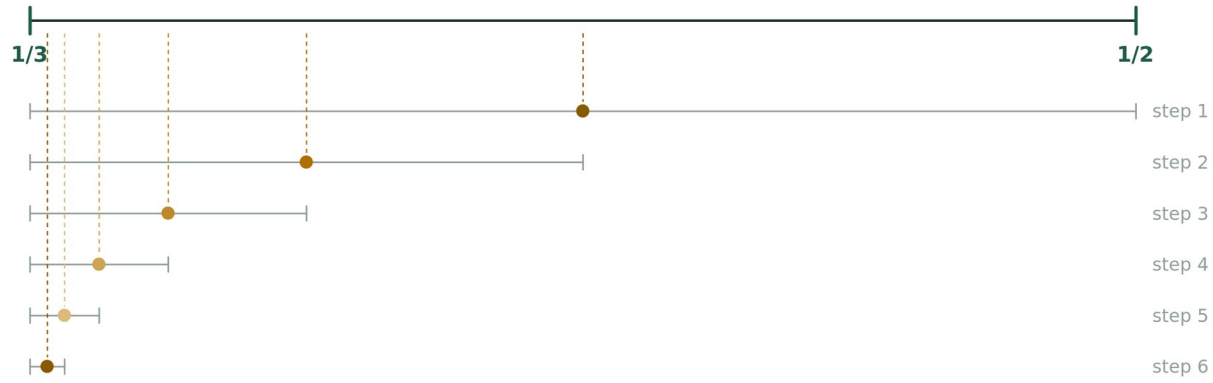
Activity 2 · Squeezing Numbers Into Gaps

Number System

Competencies: C-1.1, CG-9	Suggested time: 60 min	Team of 3–4 · Record everything

Activity 2 · Squeezing numbers into the gap (density of rationals)

Every gap you make still contains another rational number. It never runs out.



Record the gap remaining at each step. Does it ever reach zero?

Figure 2 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A software team is building a slider that must always land on a valid value. A junior developer claims that between any two allowed values there is a smallest gap. A senior developer says there is no such thing for fractions. Your team must settle it, and then produce an algorithm the developer can actually code.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Long paper strip for a number line, markers
Calculator, graph paper
Coloured pens

EXPLORE AND BUILD

1. Take two rational numbers, say 1/3 and 1/2. Find a number between them using the average. Record it.
2. Repeat: find a number between 1/3 and your new number. Record. Repeat this five more times, keeping a full record.
3. Plot each new point on your number line strip. Observe how the points crowd together.
4. Now find five rational numbers between 1/3 and 1/2 using a different method: convert both to fractions with a large common denominator.
5. Compare the two methods for speed and for how evenly spread the answers are.

6. Write the second method as a numbered algorithm any person could follow.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Step	Lower number	Upper number	Number found between	As a decimal	Gap remaining

REASON IT OUT

Q1. Did the gap ever reach zero? State clearly what the density property claims.

Q2. Which of your two methods would you code, and why? Consider both speed and predictability.

Q3. Write 0.375 and 0.3 recurring as fractions, showing the working for the recurring one.

Q4. Explain why the decimal expansion of a rational number must either terminate or repeat. A full proof is not required, but your reason should be specific.

PUSH FURTHER

Investigate which fractions terminate: test twenty fractions and sort them by whether the decimal terminates. Look at the denominators of each group in factored form and state the rule you find. Then test your rule on three new cases you did not use to find it.

ROOTS AND REACH

Decimal place value and the symbol for zero were developed in India and transmitted westward, transforming computation. Compare doing your long division in the decimal system with attempting it in Roman numerals, and describe concretely what the notation makes possible.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 3 · The Compound Interest of Small Errors

Number System

Competencies: C-1.1, CG-9	Suggested time: 65 min	Team of 3–4 · Record everything

Activity 3 · How much of π do you actually need?

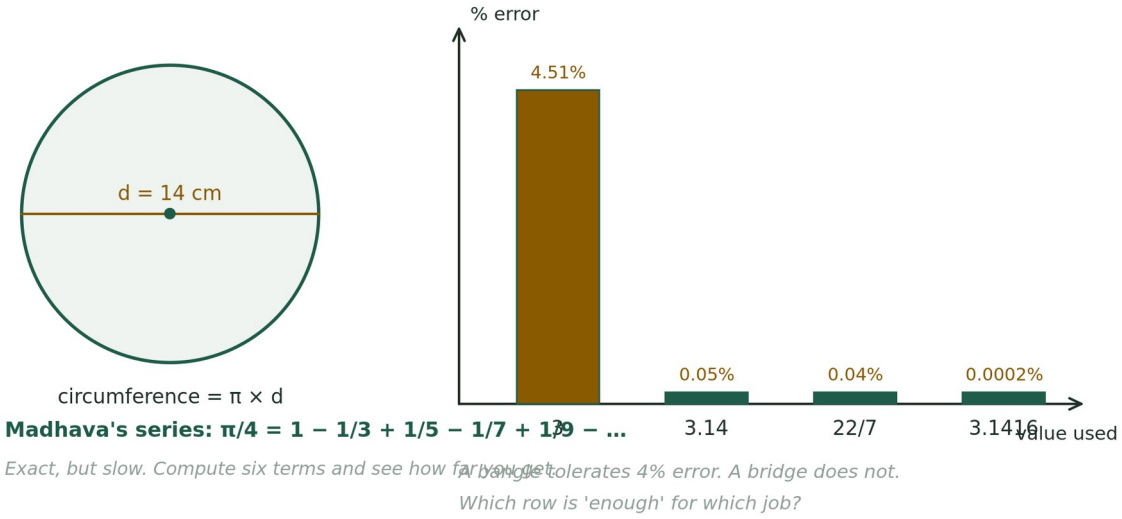

d = 14 cm
circumference = π × d
% error
4.51%
0.05%
0.04%
0.0002%
Madhava's series: π/4 = 1 − 1/3 + 1/5 − 1/7 + 1/9 − ...
3.14
22/7
3.1416
value used
Exact, but slow. Compute six terms and see how far you get.
A bangle tolerates 4% error. A bridge does not.
Which row is 'enough' for which job?
Compare Archimedes, Aryabhata and Zu Chongzhi on the same error scale.

Figure 3 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A shopkeeper rounds every bill to the nearest rupee. A customer says this is fair because errors cancel out. An accountant says rounding always favours someone. Your team will run the numbers on a real set of prices and find out — this is a study of approximation, which is what irrational numbers force upon us anyway.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

A real shopping list or bill with at least 20 items and paise values

Calculator, graph paper

Spreadsheet if available

EXPLORE AND BUILD

1. List 20 real prices with paise. Compute the exact total.
2. Round each price to the nearest rupee, then total. Record the difference from the exact total.
3. Now round every price up, and separately round every price down. Record both totals.
4. Repeat the whole exercise with the value of pi: compute the circumference of a 7 cm circle using 3, 3.14, 22/7 and a calculator value. Tabulate the errors.

5. Express each error as a percentage of the true value.
6. Identify the smallest number of decimal places that keeps the error below 0.1%.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Value of pi used	Circumference (cm)	Error (cm)	% error	Acceptable for a bangle? A bridge?

REASON IT OUT

Q1. Did the rounding errors on the bill cancel out? Give the actual figure and comment on the customer's claim.

Q2. Explain why 22/7 is a rational approximation to an irrational number, and what that means about its decimal expansion.

Q3. For which real task would 3.14 be dangerously imprecise, and for which would it be needlessly precise? Give one example of each.

Q4. How many decimal places of pi are needed to compute the circumference of the Earth to within a metre? Estimate, then check.

PUSH FURTHER

Investigate historical approximations to pi from Archimedes, Aryabhata and Zu Chongzhi. Place each on a number line, compute the error of each, and order them by accuracy. Then attempt Archimedes' method yourself: inscribe and circumscribe regular polygons in a circle and squeeze the value between the two perimeters.

ROOTS AND REACH

Madhava of Sangamagrama gave the first exact infinite series for pi. Compute the first six terms of the series pi/4 = 1 - 1/3 + 1/5 - 1/7 + … and observe how slowly it converges. Discuss the difference between an exact formula and a fast one.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

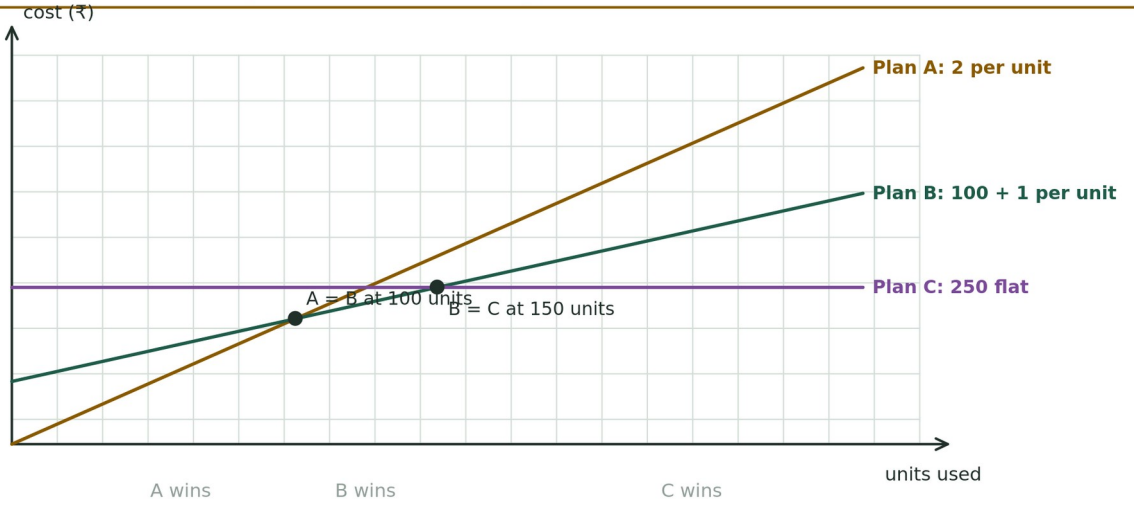
Introduction to Polynomials

Activity 4 · The Mobile Plan Comparison

Introduction to Polynomials

Competencies: C-3.2, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 4 · Three mobile plans as three straight lines



Identify the slope and the y-intercept of each line, and say what each means in rupees.

Figure 4 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Three mobile plans are advertised. Plan A: no monthly fee, 2 rupees per unit. Plan B: 100 rupees monthly, 1 rupee per unit. Plan C: 250 rupees monthly, unlimited. Every family in your neighbourhood is guessing. Your team will turn each plan into a linear polynomial and produce a chart that answers the question for any level of use.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Graph paper, ruler, coloured pens
>
> Calculator
>
> Real usage data from three volunteers, if available

EXPLORE AND BUILD

1. Write each plan as a cost expression in terms of the number of units used. Identify the constant term and the coefficient in each.

2. State the degree of each expression and explain what the degree means here.
3. Build a table of cost for 0, 50, 100, 150, 200, 250 and 300 units for all three plans.
4. Plot all three on one set of axes. Use different colours and label each line.
5. Identify the intersection points and interpret each one in plain language.
6. Divide the horizontal axis into regions and label which plan wins in each region.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Units used	Plan A cost	Plan B cost	Plan C cost	Cheapest plan

REASON IT OUT

Q1. Identify the slope and the y-intercept of each line, and state what each represents in the real situation.

Q2. Plan C is a horizontal line. What is its slope, and what does a zero slope mean here?

Q3. At exactly which usage do Plans A and B cost the same? Solve algebraically and confirm from your graph.

Q4. A family uses 180 units. Which plan should they choose, and by how much are they better off per year?

PUSH FURTHER

Model a decay situation: a phone battery loses charge at a roughly constant rate during video playback. Measure percentage charge every 5 minutes for 30 minutes, plot it, fit a straight line, and use the polynomial to predict when it hits 10%. Then test the prediction and report the error.

ROOTS AND REACH

Take the chart to three actual households, ask what plan they are on, and record whether the mathematics agrees with their choice. Where it disagrees, ask why — you will usually find a reason your model did not include.

Working Space, Sketches and Observations

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Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

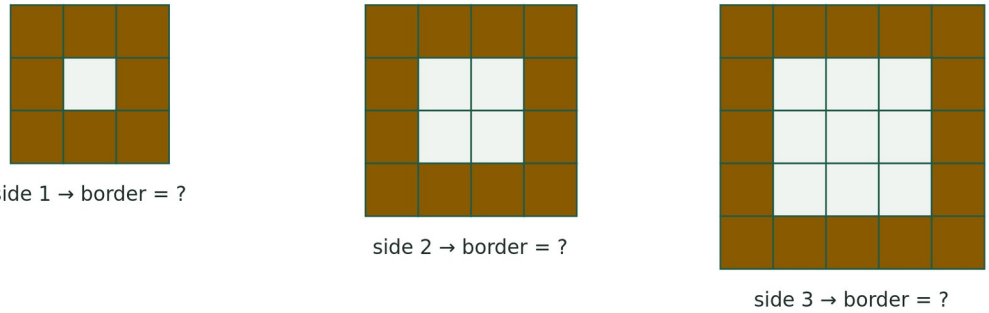
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 5 · Patterns in Growing Shapes

Introduction to Polynomials

Competencies: C-3.2, C-7.2, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 5 · The growing border — find the rule, then prove it


side 1 → border = ?
side 2 → border = ?
side 3 → border = ?
side: 1 2 3 4 5
border: ___ ___ ___ ___ ___
first differences constant?
→ what degree of rule does that mean?

Find the rule twice: from the number pattern, and from the geometry of 4 sides + 4 corners.

Figure 5 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A tile contractor builds L-shaped borders around square patios. He needs to quote tile counts instantly for any patio size. He currently counts by hand and makes errors. Your team will find the rule, express it as a polynomial, and prove it works for every size — not just the ones you tested.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Squared paper, counters, coloured pencils

Matchsticks or straws for a second pattern

Calculator

EXPLORE AND BUILD

1. Build a square patio of side 1 and draw the single-tile border around it. Count the border tiles.
2. Repeat for sides 2, 3, 4 and 5. Record the border count each time.
3. Look at the differences between consecutive counts. Are they constant? What does that suggest about the degree of the rule?
4. Write your rule as an expression in terms of the side length. Test it on side 6 by prediction, then by counting.

5. Now find the rule a second way: reason geometrically about the four sides and the four corners. Show that both expressions are equal.
6. Repeat the whole process with a matchstick pattern of joined squares in a row.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Side length	Border tiles counted	First difference	Predicted by rule	Match?

REASON IT OUT

Q1. Constant first differences indicate a linear rule. What would constant SECOND differences indicate?

Q2. Your two derivations gave the same expression. Explain why a geometric argument is more convincing than checking six cases.

Q3. Identify the degree, the terms and the coefficients of your expression.

Q4. For what side length would the contractor need exactly 100 border tiles? Solve, and comment on whether your answer is usable.

PUSH FURTHER

Extend to a pattern with constant second differences — for example, the total number of tiles in the whole patio including the border. Find the rule, identify its degree, and describe how you would recognise a quadratic pattern from a table of values alone.

ROOTS AND REACH

Give your rule to someone who has not done this activity and ask them to use it for a patio of side 12. Record any misunderstanding. Then rewrite the rule as a set of instructions in words rather than symbols, and compare which version communicates better.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

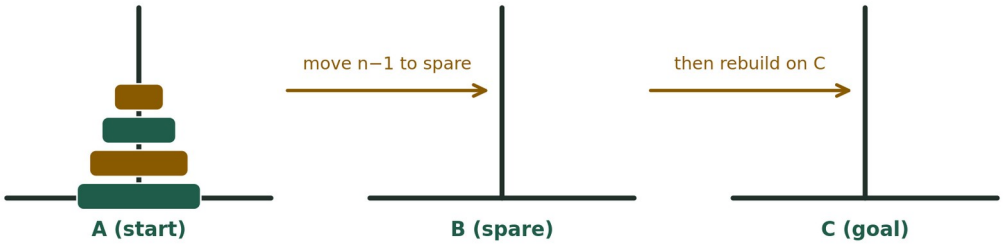
Sequences and Progressions

Activity 6 · The Tower of Hanoi and the Cost of Doubling

Sequences and Progressions

Competencies: C-8.1, CG-9	Suggested time: 80 min	Team of 3–4 · Record everything

Activity 6 · Tower of Hanoi — recursive thinking made physical



discs: 1 2 3 4 5 … 64

moves: ___ ___ ___ ___ ___

Recursive rule: M(n) = 2 × M(n−1) + 1 Explicit rule: M(n) = _____

Solve 4 discs using your 3-disc method. Write the method down before you look for a formula.

Figure 6 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A legend says monks are moving 64 golden discs between three posts, one at a time, never placing a larger on a smaller, and the world ends when they finish. Your team will find out how long that actually takes — and in doing so build the recursive thinking that underlies computer science.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Discs of different sizes: coins, bottle caps or cut cardboard circles

Three marked positions on paper

Stopwatch, calculator, graph paper

EXPLORE AND BUILD

1. Solve the puzzle with 1 disc. Record the minimum number of moves. Then 2 discs, then 3, 4 and 5.

2. Do not guess the pattern from the numbers alone. Instead, explain in words how you solved 4 discs using your 3-disc solution.
3. Turn that explanation into a recursive rule connecting the moves for n discs to the moves for n-1 discs.
4. From the recursive rule, generate the explicit rule for the number of moves.
5. Verify the explicit rule against every value you measured.
6. Compute the moves for 64 discs and, at one move per second, convert to years.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Number of discs	Minimum moves (measured)	Previous value doubled + 1	Explicit rule value	Match?

REASON IT OUT

Q1. State the difference between a recursive rule and an explicit rule, using your own two rules as the example.

Q2. Which rule is easier to use for 5 discs, and which for 64? Explain the practical trade-off.

Q3. Your move counts form a sequence. Is it an arithmetic progression, a geometric progression, or neither? Justify carefully.

Q4. Time yourself on 5 discs. Estimate the time for 10 discs from your rule, then test. Where does the estimate break down, and why?

PUSH FURTHER

Build a fractal: draw a triangle, remove the middle triangle formed by joining midpoints, and repeat on each remaining triangle for four stages. Count triangles at each stage and identify the geometric progression. Find the nth term, and compute the total shaded area as a fraction of the original.

ROOTS AND REACH

Virahanka described the sequence now widely known by another name while analysing patterns of long and short syllables in Sanskrit prosody. Work out the number of ways to arrange syllables of length 1 and 2 to total 1, 2, 3, 4, 5 and 6 beats, and identify the recursive rule you find.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

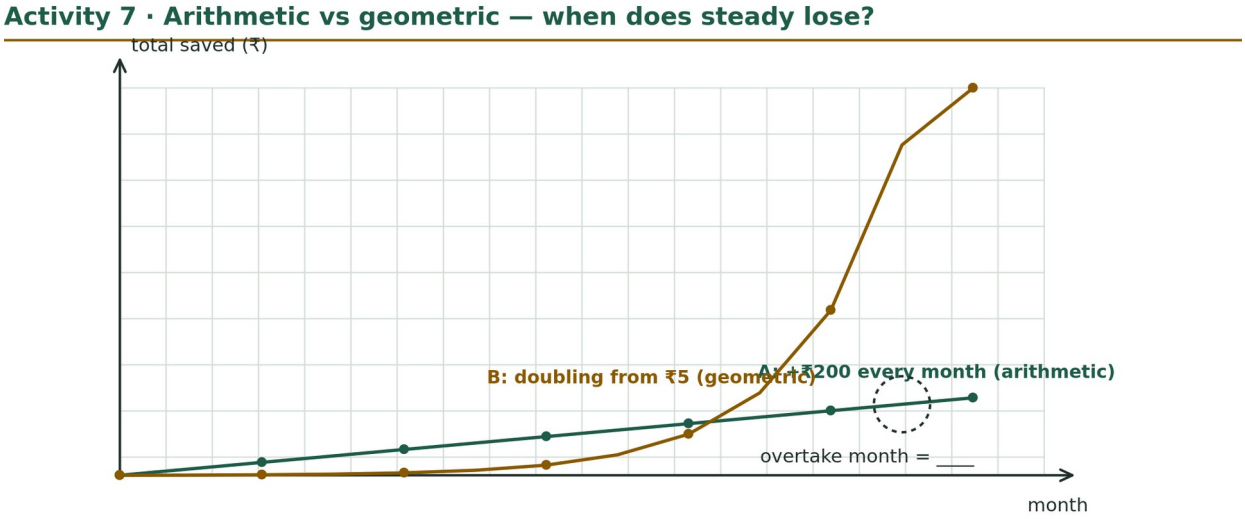
One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 7 · The Savings Race

Sequences and Progressions

Competencies: C-8.1, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything


Activity 7 · Arithmetic vs geometric — when does steady lose?
total saved (₹)
B: doubling from ₹5 (geometric)
A: +₹200 every month (arithmetic)
overtake month = ___
month

B beats A eventually no matter how large A's fixed amount is. Explain why, without formulae.

Figure 7 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Two cousins start saving on the same day. One adds a fixed 200 rupees every month. The other starts with 5 rupees and doubles the amount added each month. For a long while the first is far ahead. Your team must find exactly when the second overtakes, and explain to a sceptical parent why the answer is so counter-intuitive.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Graph paper (two sheets, one for a stretched vertical scale)
Calculator
Coloured pens

EXPLORE AND BUILD

1. Build a month-by-month table of the amount added and the running total for both cousins, for 12 months.
2. Write the explicit rule for the nth term of each sequence: the added amount and separately the running total.
3. Plot both running totals on the same graph. Note the problem with the vertical scale after month 9.
4. Redraw using a compressed or broken scale, and state clearly what you have done to the axis and why.
5. Find the exact month in which the second cousin overtakes, both from the table and from the rules.

6. Extend the second cousin's sequence to month 18 and comment on the figure.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Month	Cousin A added	Cousin A total	Cousin B added	Cousin B total	Who leads?

REASON IT OUT

Q1. Identify which sequence is arithmetic and which is geometric, and state the common difference and common ratio.

Q2. Find the sum of the first n terms for Cousin A. Derive it rather than quoting it, by pairing the first and last terms.

Q3. Explain, without formulae, why the geometric sequence is bound to overtake eventually no matter how large the fixed amount is.

Q4. Cousin B's plan is impossible in practice. At what month does it become so, and what does that tell you about the limits of a model?

PUSH FURTHER

Investigate the sum of the first n natural numbers by arranging counters into a triangle and then two triangles into a rectangle. Derive the formula from the picture. Then use the same visual trick to derive the sum of the first n odd numbers, and state what shape the answer is.

ROOTS AND REACH

Present both plans to a family member and ask which they would choose. Record their reasoning. Most people prefer the steady plan even when shown the graph — discuss why the mathematics does not settle the decision by itself.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

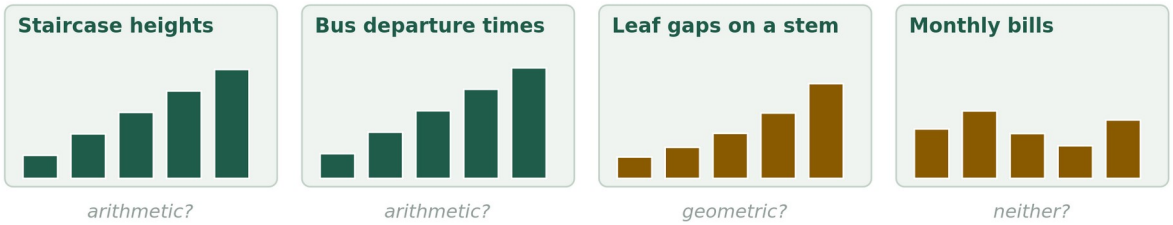
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 8 · Reading the Rule From the World

Sequences and Progressions

Competencies: C-8.1, CG-9	Suggested time: 2 sessions	Team of 3–4 · Record everything

Activity 8 · Four real sequences — classify each honestly



For each: compute FIRST DIFFERENCES and RATIOS OF CONSECUTIVE TERMS.

differences constant → arithmetic

ratios constant → geometric

neither constant → say so. Real data is allowed to be neither.

Real sequences are approximately something. State how close is close enough, and why.

Figure 8 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Sequences are usually handed to you complete. In real life you get a few noisy terms and must infer the rule. Your team will collect four real sequences from your surroundings, decide whether each is arithmetic, geometric or neither, and — where it is neither — say honestly that it is.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Measuring tape, notebook, camera
>
> Calculator, graph paper
>
> Access to a staircase, a spiral shell or plant, a bus timetable, an electricity bill

EXPLORE AND BUILD

1. Sequence 1: measure the height reached after each step on a staircase. Record ten terms.
2. Sequence 2: record bus or train departure times from a timetable for one route. Convert to minutes past the hour.
3. Sequence 3: measure the successive gaps between leaves or branches on a stem, or the widths of successive chambers in a spiral shell.
4. Sequence 4: record the last six months of a utility bill.

5. For each, compute first differences and ratios of consecutive terms. Tabulate both.
6. Classify each sequence, and where the classification is uncertain, state what further data would decide it.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Sequence	Terms 1-5	First differences	Ratios	Classification	Confidence

REASON IT OUT

Q1. Which sequence was closest to a perfect arithmetic progression, and what small deviations did you find? What caused them?

Q2. Real data is never exact. State a rule of thumb for deciding when differences are 'close enough' to constant.

Q3. For your best arithmetic sequence, predict the 20th term and state your confidence.

Q4. One sequence was probably neither type. Describe its behaviour precisely without forcing it into a category.

PUSH FURTHER

Take your utility bill sequence and attempt three different models: constant, arithmetic and geometric. Compute the predicted next value under each. Then wait for the actual next bill and see which model was closest. Report the result even if all three were wrong.

ROOTS AND REACH

Write a one-page reflection on the difference between a textbook sequence, which is exactly one thing, and a real sequence, which is approximately something. Which kind of problem do you expect to meet more often after school?

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

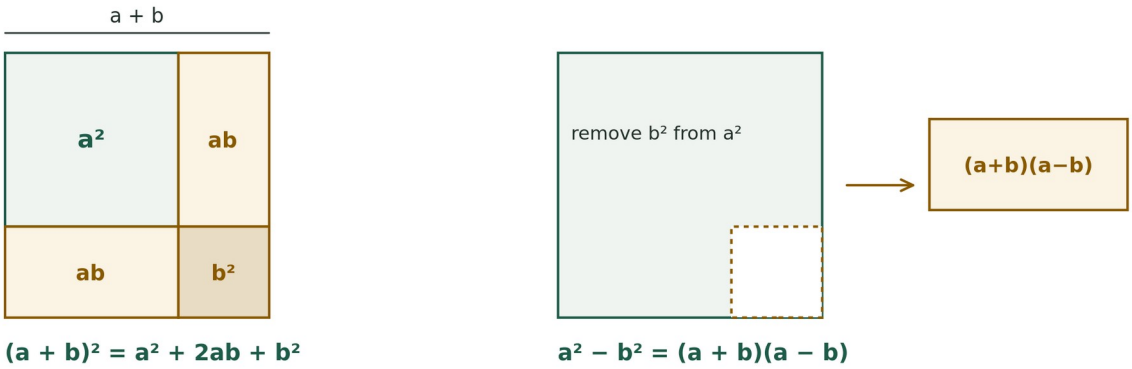
Exploring Algebraic Identities

Activity 9 · Identities You Can Hold

Exploring Algebraic Identities

Competencies: C-7.2, CG-9	Suggested time: 75 min	Team of 3–4 · Record everything

Activity 9 · Identities you can hold — area as proof



(a + b)² = a² + 2ab + b²

a² − b² = (a + b)(a − b)

Cut the pieces. Rearrange them. If the areas match, the identity is not a rule you memorised.

Does the area model still work when a < b? Check numerically and explain the gap.

Figure 9 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A student says algebra is 'just symbol pushing' with no meaning. Your team will disprove this by building every major identity as a physical rearrangement of area — so that the identity becomes something you can see is true rather than something you checked by expanding.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Card or thick paper, scissors, ruler, coloured pens
>
> Algebra tiles: large squares, small squares and rectangles cut to consistent sizes
>
> Squared paper

EXPLORE AND BUILD

1. Cut a large square of side a and a small square of side b. Cut two rectangles of sides a and b.

2. Arrange them to build a square of side (a + b). Record the total area two ways: as one square, and as the sum of the pieces. Write the identity you have just demonstrated.
3. Now demonstrate the identity for (a - b) squared by starting from the large square and removing pieces. Draw each stage.
4. Demonstrate the difference of two squares by cutting a small square from a corner of a large one and rearranging the remaining L-shape into a rectangle. State the identity and check the dimensions of your rectangle.
5. Build the identity for (a + b + c) squared using nine pieces. Record how many of each type you needed.
6. Verify each identity numerically with a = 7, b = 3 and, where needed, c = 2.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Identity	Pieces used	Area as one shape	Area as sum of pieces	Numerical check

REASON IT OUT

Q1. For the difference of two squares, state the dimensions of the rearranged rectangle and show they multiply to give the identity.

Q2. The area model requires lengths to be positive. Does the identity still hold if a is smaller than b? Check numerically and explain the gap between the model and the algebra.

Q3. Which identity was hardest to build physically, and what does that suggest about the limits of geometric proof?

Q4. Use one identity to compute 103 squared mentally. Show the steps you actually used.

PUSH FURTHER

Go the other way: given the quadratic expression x squared plus 7x plus 12, build it from tiles and rearrange into a rectangle. Read the factors off the sides. Then attempt x squared plus 5x plus 7 and explain, from the tiles, why it cannot be done with whole-number sides.

ROOTS AND REACH

Proofs without words appear across mathematical traditions, from Greek geometry to the diagrams in Indian and Chinese texts. Find one such diagram, reconstruct it with paper, and write the algebraic statement it proves.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

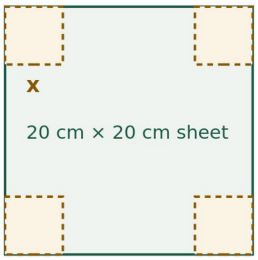
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 10 · The Packaging Optimiser

Exploring Algebraic Identities

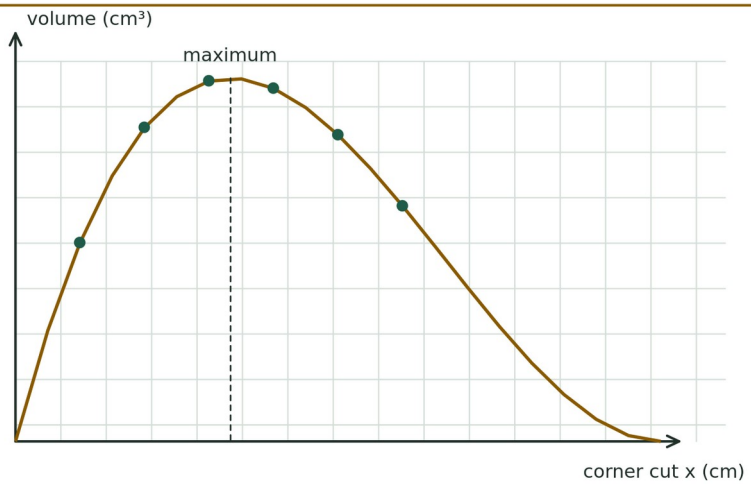
Competencies: C-7.2, C-8.3, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 10 · The open box — cut corners, then optimise



cut x from each corner, fold up the sides

volume = x (20 − 2x)²



Volume is zero at x = 0 and at x = 10. Why both?

Build six boxes. Measure with rice. Then check against your expanded expression.

Figure 10 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A cooperative packs dry snacks in open-topped boxes cut from square sheets of card. They waste material and they do not know what corner size gives the largest volume. Your team will find out — first by building, then by algebra, and you will see the two methods agree.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Square card sheets, all 20 cm by 20 cm, at least six of them

Scissors, ruler, tape

Rice or sand for volume measurement, measuring cylinder, graph paper

EXPLORE AND BUILD

1. Cut squares of side 1 cm from each corner of one sheet, fold up the sides and tape. Measure the internal dimensions and compute the volume.
2. Repeat with corner squares of 2, 3, 4, 5 and 6 cm. Compute the volume of each box.
3. Check at least two of your computed volumes by filling with rice and measuring.

4. Write an expression for the volume in terms of the corner size, using the identity for a product of three linear factors.
5. Plot volume against corner size. Sketch a smooth curve through the points.
6. Read the maximum from the graph and compare with the best box you actually built.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Corner cut (cm)	Base length (cm)	Height (cm)	Computed volume (cm^3)	Measured volume (cm^3)

REASON IT OUT

Q1. Expand your volume expression fully. What is its degree, and what does that tell you about the shape of the graph?

Q2. Your maximum from the graph probably fell between two whole numbers. How would you narrow it down experimentally?

Q3. Explain why volume falls to zero at two different corner sizes, and identify both.

Q4. The measured and computed volumes differed. List two sources of difference, one of which is not measurement error.

PUSH FURTHER

Repeat for a rectangular sheet of 30 cm by 20 cm. Predict, before doing any work, whether the optimal corner cut will be larger or smaller than before, and give a reason. Then find out. Report whether your intuition was right.

ROOTS AND REACH

Calculate the percentage of the original card wasted at your optimal cut. Then investigate whether the wasted corners could be used for anything, and propose a design that reduces waste even if it slightly reduces volume. State the trade-off numerically.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

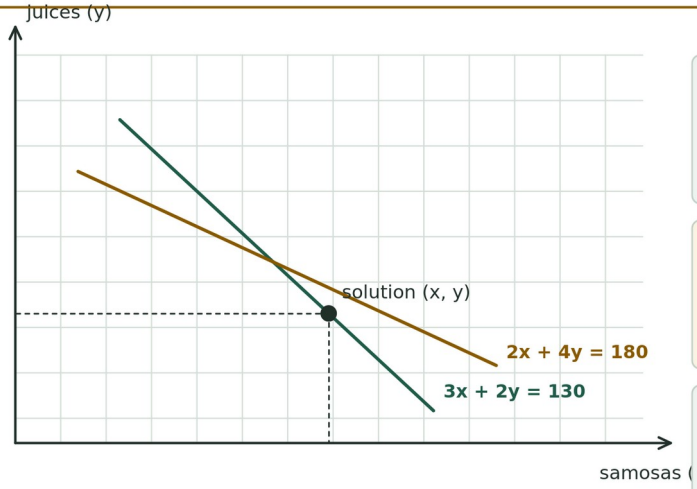
Linear Equations in Two Variables

Activity 11 · Where Two Costs Meet

Linear Equations in Two Variables

Competencies: C-3.2, C-8.1, CG-9 | **Suggested time: 75 min** | **Team of 3–4 · Record everything**

Activity 11 · Two equations, one intersection — three ways to find it



1 · Graphically
plot both lines, read the crossing.
fast to see, approximate to read.

2 · Substitution
make one variable the subject, put it into the other equation.

3 · Elimination
scale both equations so one variable cancels when added.

All three must give the same answer. If they don't, find the error — don't pick a favourite.

Figure 11 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A school canteen sells two items. On day one, 3 samosas and 2 juices cost 130 rupees; on day two, 2 samosas and 4 juices cost 180. A student wants to know the price of each without asking. Your team will solve it three ways — graphically, by substitution and by elimination — and then decide which method to teach to a younger class.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Graph paper, ruler, coloured pens
Calculator
Real price data from your canteen or a local shop

EXPLORE AND BUILD

1. Write both situations as equations in two variables. Define your variables explicitly before writing anything.
2. Solve graphically: make a table of at least three points for each equation, plot both lines, and read the intersection.

3. Solve by substitution. Show every step.
4. Solve by elimination. Show every step.
5. Compare all three answers. If any differ, find the error rather than choosing a favourite.
6. Verify your solution by substituting back into both original equations.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Method	Steps needed	Answer for item 1	Answer for item 2	Time taken	Easiest to explain?

REASON IT OUT

Q1. Which method was fastest, and which gave you the most insight into why the answer is what it is?

Q2. The graphical method gave an approximate reading. Under what circumstances would that be a problem?

Q3. Verify by substitution and show both equations balance. Why is verification not optional?

Q4. Change one number so the two lines become parallel. What does that mean about the canteen's prices?

PUSH FURTHER

Investigate the three cases: one solution, no solution, infinitely many solutions. Construct one pair of equations for each case, plot all three pairs, and state the relationship between the coefficients in each case. Then explain each case in terms of a real canteen situation.

ROOTS AND REACH

Collect two real price combinations from a shop near you, set up the equations, solve, and then ask the shopkeeper for the actual prices. Report the match or mismatch — and if it does not match, find out why (offers, rounding, or an error in your data).

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

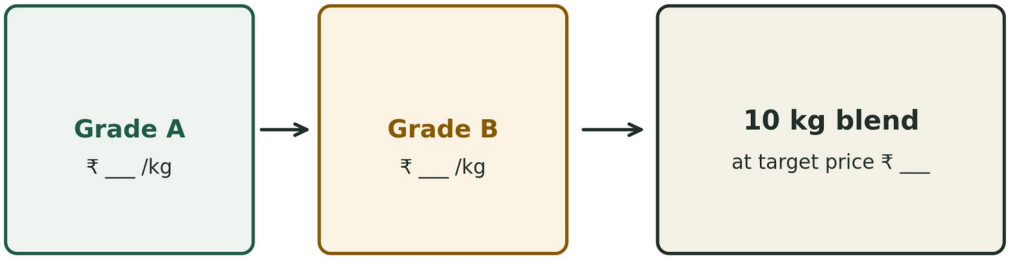
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 12 · The Mixing Problem

Linear Equations in Two Variables

Competencies: C-3.2, C-8.1	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 12 · The blending problem — two constraints, two unknowns



x + y = 10 (total mass)

a·x + b·y = 10 × target (total cost)

Solve. Then actually weigh 200 g in that ratio and measure the blend's density.

Does it match the weighted prediction? Why not?

x and y must both be non-negative. What does that say about which target prices are possible?

Figure 12 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A cooperative blends two grades of flour to hit a target price and a target protein content. Too much guesswork means loss. Your team will build the model, solve it, and then physically test the blend to see whether the mathematics survives contact with real ingredients.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Two dry materials of different densities: rice and dal, or two flours
>
> Weighing balance, containers, measuring cylinder
>
> Graph paper, calculator

EXPLORE AND BUILD

1. Set the problem: you need 10 kg of blend at a target average price, from two ingredients of known different prices. Write the two equations.
2. Solve for the quantity of each ingredient. Show both the elimination and substitution routes.
3. Scale down to a testable quantity, for example 200 g total, keeping the ratio.
4. Actually weigh and mix the two ingredients in that ratio.

5. Measure the density of each ingredient separately and of the blend. Compare the blend's measured density with a weighted prediction.
6. Record the difference and investigate it.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Ingredient	Price per kg	Mass in blend (g)	Density (g/mL)	Contribution to blend

REASON IT OUT

Q1. Set out your two equations clearly and state what each represents physically.

Q2. The measured blend density differed from the weighted prediction. Suggest a physical reason involving how the particles pack.

Q3. If one ingredient's price rises by 20%, recompute the blend. Does the ratio change a little or a lot?

Q4. Which quantity in this problem must be non-negative, and does that constrain the possible target prices?

PUSH FURTHER

Model daily-life phenomena using a table, graph and equation together: choose a real recurring cost in your home that depends on two variables, build the model, plot it, and use it to answer one question your family actually has.

ROOTS AND REACH

Take your model to whoever manages that cost at home and present the graph. Record their reaction and any factor they name that your model omitted. Add that factor as a written limitation.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

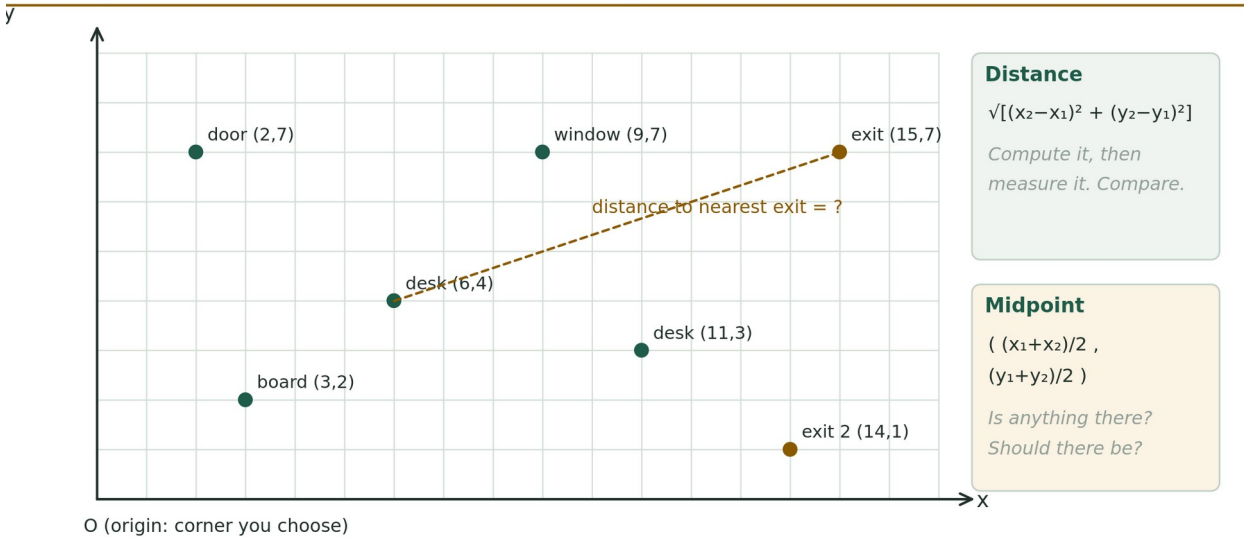
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Coordinate Geometry

Activity 13 · Mapping Your Classroom

Coordinate Geometry

Competencies: C-4.5, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything



State your scale explicitly. Most errors in this activity will be scale errors.

Figure 13 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A fire safety officer needs an accurate floor plan of your classroom with every desk, door and exit at correct relative positions, so that evacuation routes can be planned and distances computed. Hand-drawn sketches have proved useless. Your team will build a coordinate map that anyone can use.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Measuring tape, graph paper, ruler, pencil
>
> Calculator
>
> Chalk to mark an origin on the floor

EXPLORE AND BUILD

1. Choose an origin corner and two perpendicular walls as axes. State your scale explicitly, for example 1 cm on paper equals 50 cm on the floor.
2. Measure and record the coordinates of at least twelve features: desk corners, door, windows, board, exits.
3. Plot all points on graph paper. Join what needs joining.
4. Compute the distance from the farthest desk to the nearest exit using the distance formula. Then measure it directly and compare.
5. Find the midpoint of the line joining the two exits. Mark it. Is anything there? Should something be there?
6. Check whether three chosen points are collinear using coordinates, and verify visually.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Feature	x (units)	y (units)	Distance from origin	Distance to nearest exit

REASON IT OUT

Q1. Your computed and measured distances differed. State the error as a percentage and identify its main cause.

Q2. Explain the method you used to test collinearity, and why it works.

Q3. Take three desk corners and determine whether they form a right angle, using coordinates only.

Q4. Why does the choice of origin not affect any distance you computed? State the general principle.

PUSH FURTHER

Compute the area enclosed by three chosen points using coordinates. Then verify by measuring the triangle directly on the floor and computing the area from base and height. Extend to a four-sided region by splitting it into two triangles.

ROOTS AND REACH

Coordinate geometry links algebra and shape. Find one place outside mathematics class where coordinates are used daily — a spreadsheet, a seat number, a map grid, a screen pixel — and describe precisely what plays the role of the origin and the axes.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

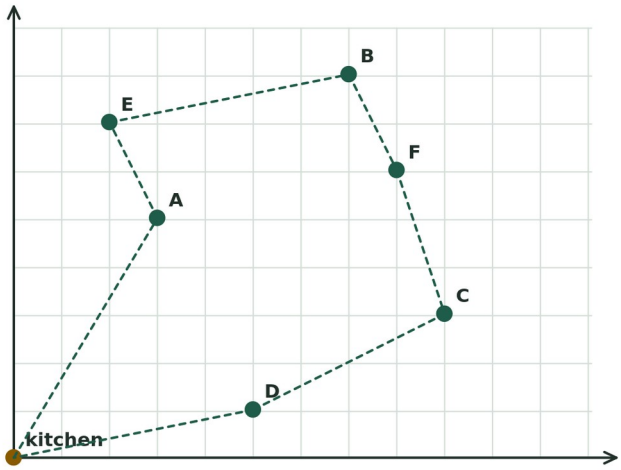
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 14 · The Delivery Route Puzzle

Coordinate Geometry

Competencies: C-4.5, C-8.3, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 14 · Routing the deliveries — straight line vs real street



dotted = straight-line model

Straight-line distance

$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$ for every pair.

Build the full distance table, then try three different orders and total each.

6 deliveries → how many possible orders?

Then overlay the real streets

Roads are not straight lines. Recompute using paths that actually exist and find the pair with the biggest difference.

What feature of the area explains it?

You will not check every order by hand. Say so — that honesty is part of the answer.

Figure 14 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A tiffin service delivers to six addresses from one kitchen. The current route was decided by habit. Your team will place all points on a coordinate grid, compute distances, and find a shorter route — then confront the fact that the shortest straight-line route may not be the shortest real route.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

A local map or a grid drawn to represent your area
Graph paper, ruler, string, calculator
Coloured pens

EXPLORE AND BUILD

1. Place the kitchen at the origin. Assign coordinates to six delivery points using a stated scale.
2. Compute the straight-line distance from the kitchen to each point.
3. Compute the distance between every pair of delivery points. Record in a table.
4. Find the total length of the current route (in the order given) and of at least three alternative orders.
5. Identify the shortest order you can find. State honestly that you have not checked every possibility.

6. Now overlay the actual streets. Recompute using only paths that exist, and compare.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

From	To	Straight-line distance	Actual road distance	Difference

REASON IT OUT

Q1. By how much did your best route improve on the original? Express it as a percentage.

Q2. Straight-line and road distances differed most for which pair? What feature of the area explains it?

Q3. How many possible orders are there for six deliveries? Compute it, and comment on whether checking all of them by hand is realistic.

Q4. Compute the midpoint of the two farthest delivery points. Would that be a sensible place for a second kitchen? Give one reason for and one against.

PUSH FURTHER

Add a constraint: one delivery must be made first because the food is hot. Re-optimise under that constraint and report how much the constraint costs in extra distance. Then add a second constraint and observe how quickly the problem becomes harder.

ROOTS AND REACH

Break the routing problem into sub-problems and write your method as a numbered algorithm. Give it to another team with a different set of six points and check whether they reach a good route by following your steps alone.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

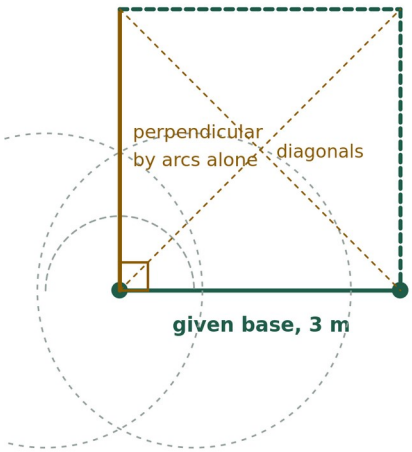
Euclid's Geometry: Axioms and Postulates

Activity 15 · Building a Perfect Square With Rope

Euclid's Geometry: Axioms and Postulates

Competencies: C-7.1, C-7.3	Suggested time: 75 min	Team of 3–4 · Record everything

Activity 15 · Laying out a square with rope only (Sulbasutra method)



The test — no protractor allowed

Measure both diagonals. Equal?

Measure all four sides. Equal?

Equal sides alone do NOT make a square.
Equal sides AND equal diagonals do.

Then double it

Construct a square of exactly twice the area. Its side cannot be twice as long — it is the old side × √2.

Back to Activity 1.

Write down which axiom or postulate justified each step you took.

Figure 15 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A mason must lay out a square foundation on open ground. He has no set square and no protractor — only rope, pegs and a straight edge. This is exactly the problem the Sulbasutras solved for altar construction. Your team will do it on the school ground and prove that the result really is a square.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Long rope or string (at least 10 m), pegs or sticks, chalk

Measuring tape

Open ground or a large floor area

EXPLORE AND BUILD

1. Mark a straight base line of 3 m with two pegs. This is your given side.

2. Using only rope lengths and arcs, construct a perpendicular at one end. Record the exact sequence of steps you used.
3. Complete the four corners. Do not measure any angle at any point.
4. Now test your square: measure both diagonals. Measure all four sides.
5. Record how far your figure deviates from a perfect square, in centimetres.
6. Write out which axioms and postulates you relied on at each construction step.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Measurement	Value 1	Value 2	Difference	Acceptable?

REASON IT OUT

Q1. State the five postulates of Euclidean geometry. Identify which ones your construction used and where.

Q2. Your two diagonals were nearly equal. Explain why equal diagonals plus equal sides guarantees a square, and why equal sides alone does not.

Q3. Which of your steps was an axiom (a general truth) and which was a postulate (a geometric assumption)? Give one example of each from your own work.

Q4. Define parallelism of straight lines. Where in your construction did you rely on it?

PUSH FURTHER

Construct a square of exactly twice the area of your first square, using rope and pegs only. State your method, then verify by measuring. Explain why the side of the new square cannot be exactly twice the old side, connecting back to the irrationality work in Activity 1.

ROOTS AND REACH

Geometry grew from the practical needs of building, land measurement and ritual construction in Egypt, Mesopotamia, India and Greece. Choose one civilisation, find one specific geometric problem it needed to solve, and describe how the need shaped the mathematics.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 16 · What Counts as a Proof?

Euclid's Geometry: Axioms and Postulates

Competencies: C-7.1, C-7.3	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 16 · What counts as a proof? Rank these four claims

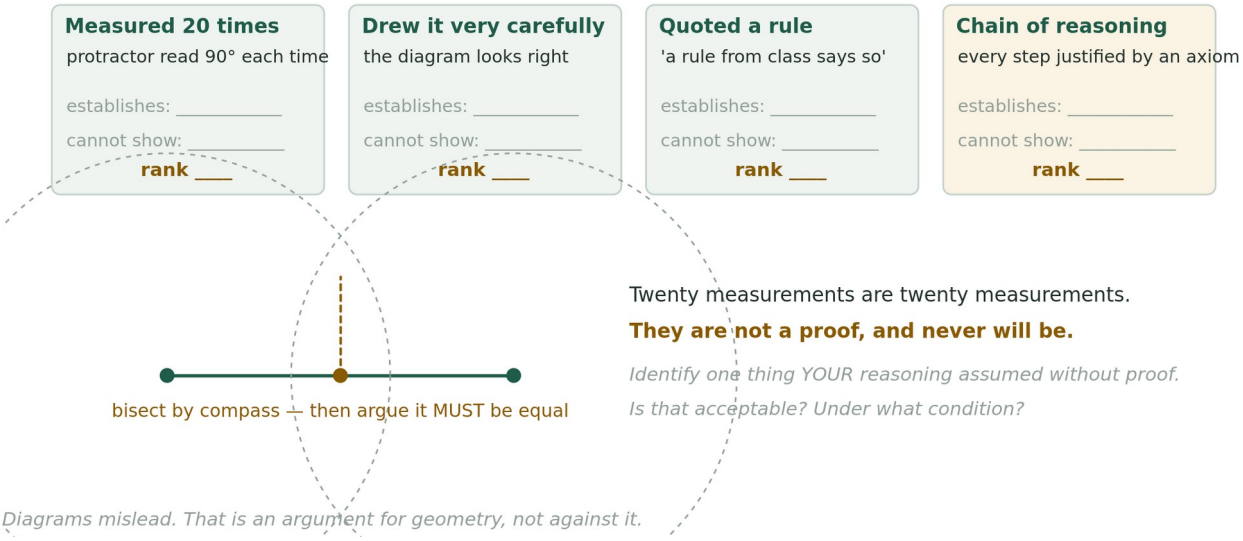

Measured 20 times
protractor read 90° each time
establishes: _____
cannot show: _____
rank ___
Drew it very carefully
the diagram looks right
establishes: _____
cannot show: _____
rank ___
Quoted a rule
'a rule from class says so'
establishes: _____
cannot show: _____
rank ___
Chain of reasoning
every step justified by an axiom
establishes: _____
cannot show: _____
rank ___
bisect by compass — then argue it MUST be equal
Twenty measurements are twenty measurements.
They are not a proof, and never will be.
Identify one thing YOUR reasoning assumed without proof.
Is that acceptable? Under what condition?
Diagrams mislead. That is an argument for geometry, not against it.

Figure 16 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Four students each claim to have proved that a certain construction gives a right angle. One measured with a protractor twenty times. One drew a very careful diagram. One quoted a rule from memory. One gave a chain of reasoning from stated assumptions. Your team must rank these as proofs and defend the ranking.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Compass, ruler, protractor, paper
> Cards with the four arguments written out
> Coloured pens

EXPLORE AND BUILD

1. Carry out a construction: bisect a given line segment using compass and straight edge only.
2. Measure both halves with a ruler. Record the readings and the difference.
3. Now write a chain of reasoning showing the halves MUST be equal, referring only to what you constructed.
4. Repeat the exercise for bisecting an angle.

5. For each of the four student claims, write one sentence on what it establishes and one on what it fails to establish.
6. Rank them and justify the ranking to another team.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Claim type	What it shows	What it cannot show	Rank	Reason

REASON IT OUT

Q1. Measurement twenty times still does not prove equality. Explain precisely why not.

Q2. A diagram can mislead. Give an example of a property that looks true in a drawing but is not.

Q3. Identify one statement in your own chain of reasoning that you assumed without proof. Is that acceptable? Under what condition?

Q4. Explain the role of definitions in a proof, using 'bisect' as your example.

PUSH FURTHER

Construct a figure that appears to show two clearly unequal lengths as equal (a standard optical illusion). Measure both. Write a short piece explaining why this is not an argument against geometry but an argument for it.

ROOTS AND REACH

Justify one of your constructions using only the axioms of measurement and the rules for geometric objects, writing each step on a separate line with its justification beside it. Hand it to another team to find any unjustified leap.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

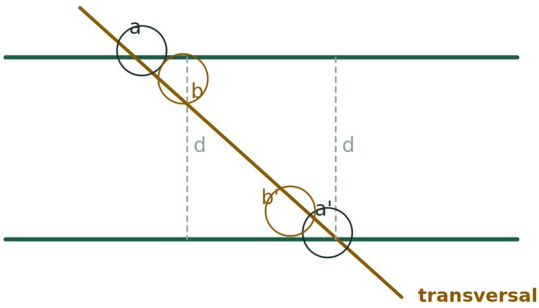
Lines and Angles

Activity 17 · The Parallel Test on Real Structures

Lines and Angles

Competencies: C-7.1, C-7.3, CG-9 | **Suggested time: 70 min** | **Team of 3–4 · Record everything**

Activity 17 · Three independent tests for parallel lines



Apply all three to five real structures

- railings
- floor tiles
- window frames
- road markings
- shelf edges

Where the three tests disagree, find out why before deciding which one to trust.

Test 1 · equal perpendicular distance at three separate points

Test 2 · corresponding angles a = a'

Test 3 · alternate interior angles b = b'

How large a deviation are you willing to call 'parallel'? State the threshold and defend it.

Figure 17 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A carpenter is fitting shelves and insists they are parallel. A student with a protractor disagrees. Rather than argue, your team will develop three independent tests for parallelism, apply them to real structures around the school, and see whether the tests agree.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Protractor, ruler, set square, string

Chalk, camera

Access to railings, tiles, window frames, road markings

EXPLORE AND BUILD

1. Test 1 — equal distance: measure the perpendicular distance between two lines at three separated points.
2. Test 2 — corresponding angles: draw or identify a transversal and measure the corresponding angles.

3. Test 3 — alternate angles: on the same transversal, measure the alternate interior angles.
4. Apply all three tests to five real pairs of lines around the school. Record every reading.
5. Where the three tests disagree, investigate why before deciding which to trust.
6. Record the largest deviation you found and where.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Structure	Distance test (3 readings)	Corresponding angles	Alternate angles	Parallel? Y/N

REASON IT OUT

Q1. State and apply the linear pair theorem to one of your measured transversals, showing the arithmetic.

Q2. Prove that vertically opposite angles are equal, using a linear pair argument. Set it out in numbered steps.

Q3. Explain transitivity of parallelism and give a real example from your measurements.

Q4. Your readings had a spread. State how large a deviation you are willing to call 'parallel', and justify that threshold.

PUSH FURTHER

Prove by contradiction that a triangle cannot have two obtuse angles. Then prove that every triangle has at least two acute angles. Then investigate: can a triangle have all three angles less than 60 degrees? Explain the result and connect it to the angle sum.

ROOTS AND REACH

Break one of your proofs into ordered steps, each with the axiom or theorem that justifies it. Then remove one step and give it to another team — see whether they notice the gap. Report what that tells you about how proofs are actually read.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

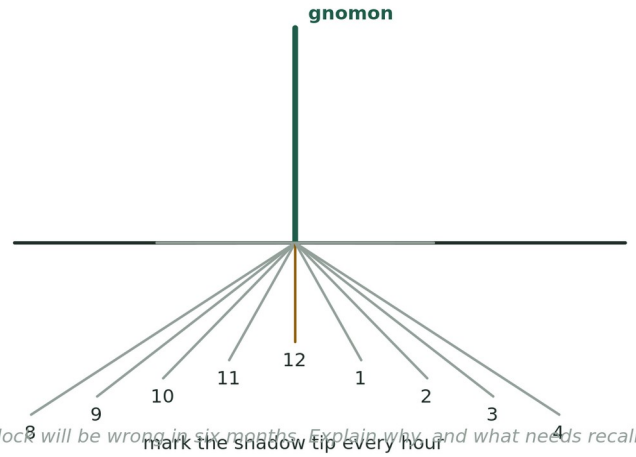
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 18 · The Shadow Clock

Lines and Angles

Competencies: C-7.1, CG-9	Suggested time: 1 day	Team of 3–4 · Record everything

Activity 18 · The shadow clock — angles with real consequences



Measure and classify

angle between consecutive lines

acute / right / obtuse / reflex

Are the hourly angles equal?
If not, what does that tell you?

Then find true north

The shortest shadow points along the north–south line. Compare with a magnetic compass and measure the gap.

What is that gap called?

Your clock will be wrong in six months. Explain why, and what needs recalibrating.

Figure 18 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Before clocks, angles told the time. Your team will build a working shadow clock, calibrate it against a real clock, and in doing so measure and classify a large number of angles with real consequences for accuracy.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

A vertical stick (gnomon), flat ground or a board, chalk
Protractor, measuring tape, watch
Large sheet of paper or card

EXPLORE AND BUILD

1. Fix a vertical stick on level ground in an unshaded spot. Check verticality carefully — state how.
2. Every hour from 8 a.m. to 4 p.m., mark the shadow tip and draw the shadow line. Note the clock time.
3. Measure the angle between consecutive shadow lines. Record all of them.
4. Measure the shadow length at each hour and tabulate.
5. Classify each angle you measured as acute, right, obtuse or reflex.
6. Identify the hour at which the shadow was shortest and note the direction it pointed.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Time	Shadow length (cm)	Angle from previous line (°)	Angle type	Direction

REASON IT OUT

Q1. Were the hourly angles equal? If not, explain what that means about the relationship between shadow angle and time.

Q2. A straight angle is two right angles and measures 180 degrees. Where in your shadow diagram does a straight angle appear?

Q3. Identify a pair of vertically opposite angles in your figure and verify they are equal by measurement.

Q4. Your clock will be wrong in six months. Explain why, and state what would need recalibration.

PUSH FURTHER

Use the shortest shadow to determine true north, then compare with a magnetic compass. Measure and report the difference. Investigate what that difference is called and why it exists.

ROOTS AND REACH

Sundials and shadow measurement were used across Indian astronomy for calendar-making and for determining latitude. Investigate one instrument at a Jantar Mantar observatory, describe its geometry, and explain what quantity it was designed to measure.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

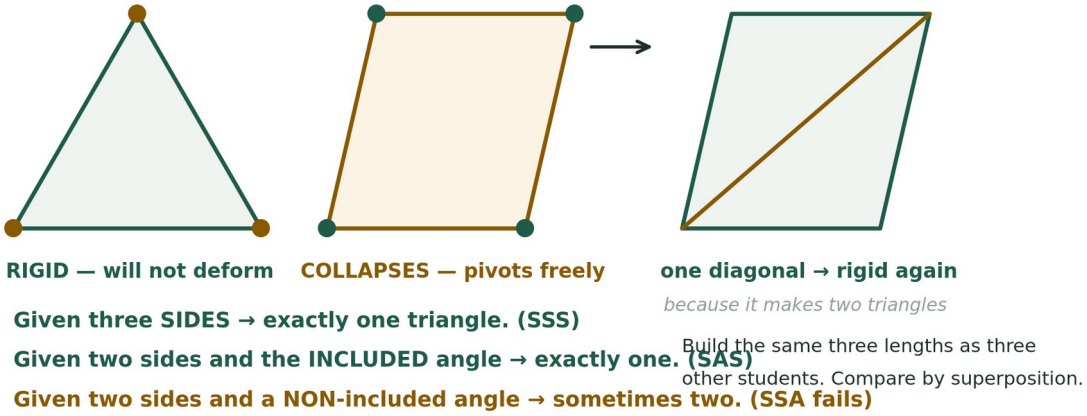
Triangles: Congruence Theorems

Activity 19 · Why Bridges Are Made of Triangles

Triangles: Congruence Theorems

Competencies: C-4.1, C-7.3	Suggested time: 75 min	Team of 3–4 · Record everything

Activity 19 · Why every bridge is made of triangles


RIGID — will not deform
COLLAPSES — pivots freely
one diagonal → rigid again
because it makes two triangles
Given three SIDES → exactly one triangle. (SSS)
Given two sides and the INCLUDED angle → exactly one. (SAS)
Given two sides and a NON-included angle → sometimes two. (SSA fails)
Build the same three lengths as three other students. Compare by superposition.

Rigidity is a physical fact. Congruence is the mathematical reason for it.

Figure 19 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A student asks why every bridge, crane, roof truss and bicycle frame is full of triangles rather than the squares that are easier to build. Your team will demonstrate rigidity physically, quantify it, and connect it directly to the congruence conditions.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Strips of card or ice-cream sticks, paper fasteners or pins
> Weights, ruler, protractor
> Thread, scissors

EXPLORE AND BUILD

1. Build a triangle and a quadrilateral from strips joined loosely at the corners so they can pivot.
2. Try to deform each. Record what happens and measure the deformation.

3. Add one diagonal strip to the quadrilateral. Test again and record.
4. Load both shapes with identical weights and measure the deflection.
5. Now the mathematics: given three side lengths, build a triangle. Ask three other students to build one from the same three lengths. Compare all four by superposition.
6. Repeat the experiment giving only two sides and the included angle, then only two sides and a non-included angle.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Given information	Number of different shapes possible	Congruence condition?	Evidence

REASON IT OUT

Q1. Three sides determined the triangle uniquely. Name the congruence condition and state it precisely.

Q2. Two sides and a non-included angle allowed more than one triangle in your test. Draw both and explain why SSA generally fails.

Q3. Identify one situation where SSA IS a valid congruence condition, and state the extra requirement.

Q4. Explain what correspondence between vertices means, and why the order of letters in a congruence statement matters.

PUSH FURTHER

Prove the basic property of an isosceles triangle: that the angles opposite the equal sides are equal. Construct the proof using a congruence condition, setting out the auxiliary line you draw and justifying why you may draw it.

ROOTS AND REACH

Count the triangles in a real structure near you — a gate, a roof, a tower, a bicycle. Photograph it and mark the triangles. Then identify one place where the designer used a quadrilateral anyway, and work out why that was acceptable there.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 20 · Converses That Fail

Triangles: Congruence Theorems

Competencies: C-4.1, C-7.3	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 20 · Converses that fail — one counterexample is enough

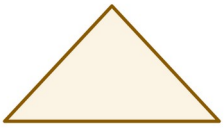
Statement
If a triangle is equilateral,
then it is isosceles.

TRUE

Its converse
If a triangle is isosceles,
then it is equilateral.

FALSE

equilateral



isosceles but NOT equilateral

← this single figure destroys the converse

Swapping P and Q is mechanical. The truth value does not come along with it.

A thousand examples establish nothing. One counterexample settles everything.

Figure 20 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A student writes: 'If a triangle is equilateral, it is isosceles. Therefore if a triangle is isosceles, it is equilateral.' The reasoning feels symmetric and is wrong. Your team will build a collection of statements and their converses, sort them into true and false, and find counterexamples for the false ones.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
> Paper, ruler, compass, protractor
> Cards for statements
> Coloured pens

EXPLORE AND BUILD

1. Write ten geometric statements of the form 'if P then Q' drawn from what you have studied.
2. For each, write its converse carefully. Note that swapping P and Q is a mechanical operation but the truth value is not.
3. For each converse, decide whether it is true. Where you claim it is false, you must produce and draw a counterexample.
4. Where you claim it is true, sketch the argument — you need not give a full proof.

5. Sort all twenty statements into a four-way table: statement true or false, converse true or false.
6. Find at least one pair where both the statement and its converse are true, and note what that means.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Statement	Converse	Statement true?	Converse true?	Counterexample if false

REASON IT OUT

Q1. Explain the notion of a proposition and of a converse in your own words, using one of your examples.

Q2. One counterexample is enough to destroy a claim. Explain why, and why a thousand examples are not enough to establish one.

Q3. Find a pair where statement and converse are both true. What is the standard way of writing such a pair as a single statement?

Q4. Diagram accuracy influenced your judgement at some point. Describe where, and how you guarded against being misled.

PUSH FURTHER

Take the statement 'if two triangles are congruent then their areas are equal'. Write its converse, decide whether it is true, and produce either a proof sketch or a counterexample. Then do the same for perimeters.

ROOTS AND REACH

Find a converse error in everyday reasoning — in an advertisement, a news claim, or a common saying. Write out the original statement, the converse, and explain exactly what goes wrong.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

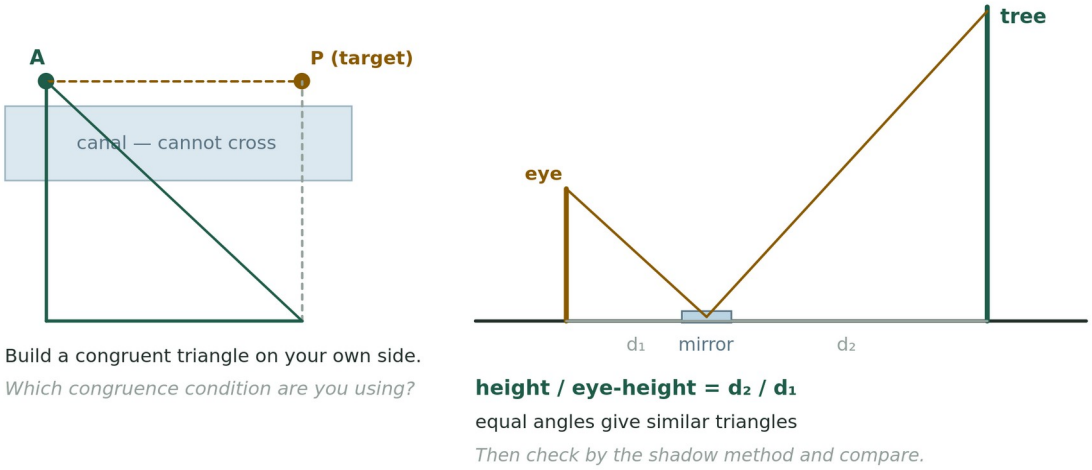
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 21 · Measuring What You Cannot Reach

Triangles: Congruence Theorems

Competencies: C-4.1, C-7.3, C-8.1	Suggested time: 80 min	Team of 3–4 · Record everything

Activity 21 · Measuring what you cannot reach



Where possible, measure one target directly and report your percentage error.

Figure 21 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

You must find the width of a canal, the height of a tree, and the distance to a building across a road — without crossing, climbing or trespassing. Surveyors have done this for centuries using congruent triangles. Your team will do all three and check every answer against a direct measurement where possible.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Measuring tape, pegs, string, ruler

A mirror, a straight stick, chalk

Notebook, calculator

EXPLORE AND BUILD

1. Inaccessible width: set up a triangle on your side of the obstacle that is congruent to a triangle spanning it. Draw your set-up clearly before measuring.
2. Take all necessary measurements and compute the inaccessible distance. State which congruence condition your method relies on.

3. Tree height by mirror: place a mirror flat on the ground, step back until you see the treetop in it, and measure the two ground distances and your eye height. Compute.
4. Tree height by shadow: measure the tree's shadow and a metre stick's shadow at the same time. Compute.
5. Compare the two tree-height results. Investigate any difference.
6. Where possible, measure one of your three targets directly and compute the percentage error.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Target	Method used	Measurements taken	Computed value	Direct value	% error

REASON IT OUT

Q1. For the canal method, name the congruence condition used and identify exactly which sides and angles you matched.

Q2. The mirror method uses reflection. Explain why it produces two triangles with equal corresponding angles.

Q3. Which of your three methods was most accurate, and which was most practical? They may not be the same.

Q4. List every assumption in the shadow method. Which one is most likely to be violated in practice?

PUSH FURTHER

Design a fourth method of your own for a target of your choosing. Before measuring, write down the geometric principle it relies on and predict its accuracy. Then test it and report whether your prediction held.

ROOTS AND REACH

Triangle rigidity gives strength; triangle congruence gives measurement. Write a short piece on how one geometric fact serves two entirely different practical purposes, and find one more application of triangles that fits neither category.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

4-gons (Quadrilaterals)

Activity 22 · The Tiling Floor

4-gons (Quadrilaterals)

Competencies: C-4.2, C-7.3	Suggested time: 75 min	Team of 3–4 · Record everything

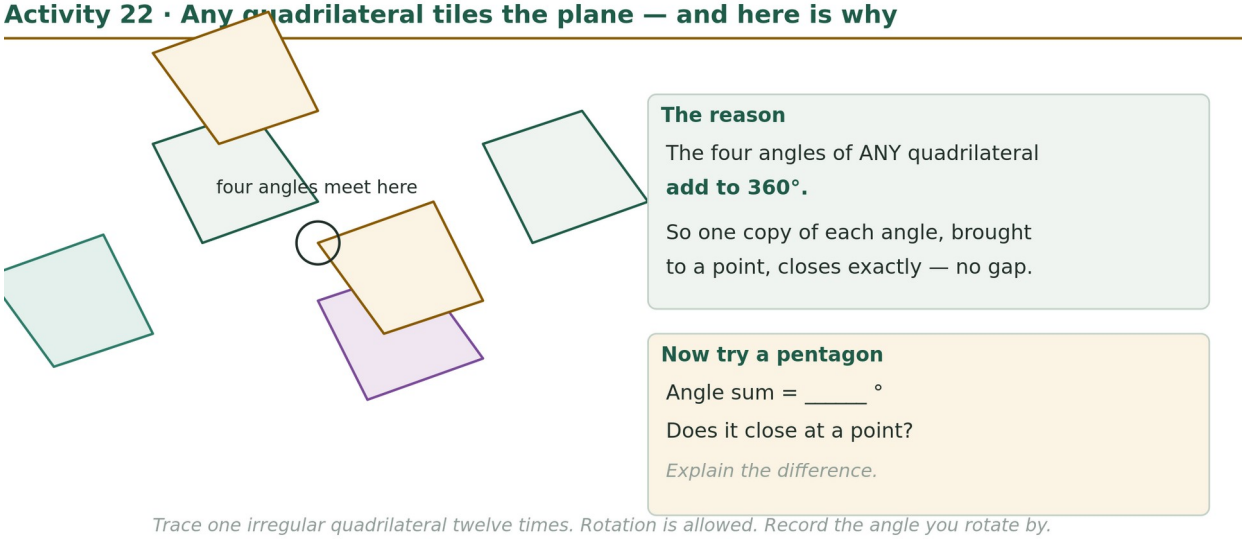

Activity 22 · Any quadrilateral tiles the plane — and here is why
four angles meet here
The reason
The four angles of ANY quadrilateral
add to 360°.
So one copy of each angle, brought
to a point, closes exactly — no gap.
Now try a pentagon
Angle sum = _____ °
Does it close at a point?
Explain the difference.
Trace one irregular quadrilateral twelve times. Rotation is allowed. Record the angle you rotate by.
Then join the midpoints of your quadrilateral's sides. What shape appears, and why?

Figure 22 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A tile-layer claims that ANY four-sided tile, however irregular, can tile a floor with no gaps. His apprentice thinks only regular shapes work. Your team will settle it by cutting irregular quadrilaterals and attempting the tiling — and then explain the result with the angle sum.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Card, scissors, ruler, protractor
> Coloured pens, large sheet of paper
> Tracing paper

EXPLORE AND BUILD

1. Cut one irregular quadrilateral. Trace around it to make at least twelve identical copies.
2. Measure all four angles of your quadrilateral and record the sum.

3. Attempt to tile a region with your copies, allowing rotation. Record whether you succeed and how you had to arrange them.
4. Look at each meeting point of tiles. Count how many tiles meet and which angles come together there.
5. Repeat with a second, very different irregular quadrilateral.
6. Attempt the same with an irregular pentagon and record what happens.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Shape	Angle 1	Angle 2	Angle 3	Angle 4	Sum	Tiles?

REASON IT OUT

Q1. Explain, using the angle sum, why four copies of any quadrilateral can meet at a point with no gap.

Q2. Frame a precise definition of a four-sided figure. Does your definition exclude a crossed figure? Should it?

Q3. Your arrangement required rotating tiles. State the angle of rotation used and connect it to central symmetry.

Q4. Why did the pentagon behave differently? Compute its angle sum and explain.

PUSH FURTHER

Investigate the midpoint figure: join the midpoints of the four sides of your irregular quadrilateral. Measure the resulting figure's sides and angles. State what you find, then prove it using the midpoint theorem.

ROOTS AND REACH

Find a real tiled surface near you with a non-square repeating unit. Photograph it, identify the repeating shape, and determine what symmetry operations map the pattern onto itself.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

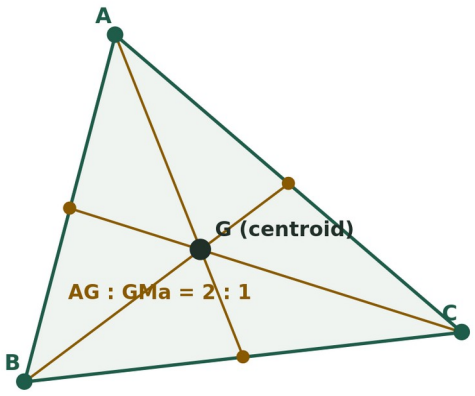
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 23 · Balancing a Triangle on a Pin

4-gons (Quadrilaterals)

Competencies: C-4.2, C-7.3	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 23 · Medians, the centroid, and the 2 : 1 ratio



Three lines meeting at one point is not automatic

Draw all three medians on an acute, an obtuse and a right-angled triangle.

Measure the closest approach of the three.

Then prove concurrency with the midpoint theorem.

Test it physically

Cut the triangle from card. Balance it on a pin at G.

Then hang it from a vertex with a plumb line, twice. The verticals cross at G.

Midpoint theorem: the segment joining two midpoints is parallel to the third side and half its length.

Figure 23 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A mobile-maker needs to hang irregular triangular pieces so they balance perfectly. She has been finding the point by trial and error. Your team will find it geometrically, prove why it works, and show that three separate lines all pass through one point — a fact that is not obvious and needs proof.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Card, scissors, ruler, pencil, pin or needle

Thread, small weight for a plumb line

Protractor

EXPLORE AND BUILD

1. Cut three different triangles from card: acute, obtuse and right-angled.
2. For each, construct the three medians by joining each vertex to the midpoint of the opposite side.
3. Observe whether all three medians pass through a single point. Measure how far apart they are at the closest approach.

4. Measure the distance from each vertex to the intersection point, and from the intersection to the opposite midpoint. Compute the ratio.
5. Test physically: balance each triangle on a pin at the intersection point. Record the result.
6. Also hang each triangle from a vertex with a plumb line and mark the vertical; repeat from another vertex and see where the lines cross.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Triangle	Vertex-to-point distance	Point-to-midpoint distance	Ratio	Balanced?

REASON IT OUT

Q1. What ratio did you find, and did it hold for all three triangles including the obtuse one?

Q2. Concurrency of three lines is not automatic. Sketch the proof that the medians are concurrent, using the midpoint theorem.

Q3. Prove the midpoint theorem: the segment joining midpoints of two sides is parallel to the third side and half its length.

Q4. Your plumb-line method and your median method gave the same point. Explain the physical reason.

PUSH FURTHER

Given three vertices of a parallelogram by coordinates, find the fourth. Then find the midpoint of each diagonal and show they coincide. State what this proves about parallelograms and connect it to central symmetry.

ROOTS AND REACH

Prove one characterisation of a parallelogram of your choice — for example, that a quadrilateral whose diagonals bisect each other is a parallelogram. Then form its converse and decide whether that is also true.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

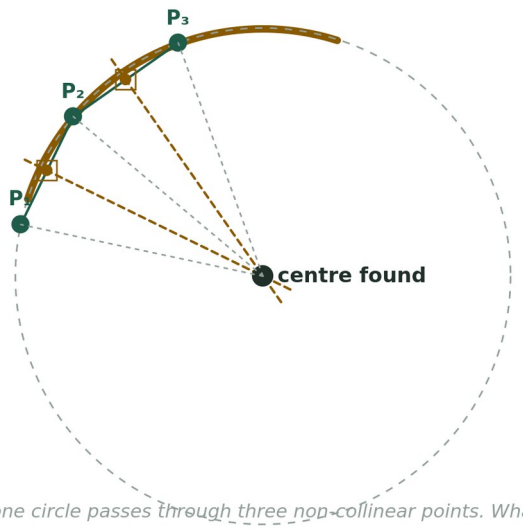
Circles

Activity 24 · Finding the Centre of a Broken Wheel

Circles

Competencies: C-7.3, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 24 · Finding the centre of a broken wheel



Why it works

Every point on the perpendicular bisector of a chord is equidistant from the chord's two endpoints — **so the centre must lie on it.**

Test the method's limits

Try three points very close together. Then three spread far apart.

Which gave a better centre? Explain how a small error in a nearly-flat chord swings the bisector a long way.

Exactly one circle passes through three non-collinear points. What if they are collinear?

Figure 24 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

An archaeologist has a fragment of a stone wheel — less than a third of the rim survives. To catalogue it she needs the original diameter. There is no centre to measure from. Your team must reconstruct the centre from the arc alone, and prove the method is valid.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Card circles, some cut into arcs; a plate or bowl rim
Compass, ruler, set square, pencil
String

EXPLORE AND BUILD

1. Take an arc fragment. Mark any three points on it.

2. Construct the perpendicular bisector of the chord joining the first two points, and of the chord joining the second and third.
3. Mark their intersection. Claim: this is the centre. Test it by measuring the distance to all three original points.
4. Draw the full circle and compare with the original if you have it.
5. Repeat with three points chosen very close together, and again with three points spread far apart. Compare accuracy.
6. Repeat the whole method on a real curved object such as a plate rim or a wheel.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Trial	Point spacing	Distance to P1	Distance to P2	Distance to P3	Spread

REASON IT OUT

Q1. Explain why the perpendicular bisector of a chord must pass through the centre. Give the argument, not just the statement.

Q2. Why must the three points be non-collinear? What happens if they are nearly collinear?

Q3. Closely spaced points gave a worse result. Explain why, thinking about how a small measuring error affects the bisector direction.

Q4. Explain why exactly one circle passes through three non-collinear points.

PUSH FURTHER

Construct the circumcircle of an acute triangle, an obtuse triangle and a right-angled triangle. Record where the circumcentre lies in each case, and state the general rule you find. For the right-angled case, note where the centre falls and connect it to the angle in a semicircle.

ROOTS AND REACH

Circular wheels changed transport, farming, pottery and machinery. Choose one and describe specifically what geometric property of the circle makes it work — and what would fail if the shape were a regular polygon with many sides instead.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

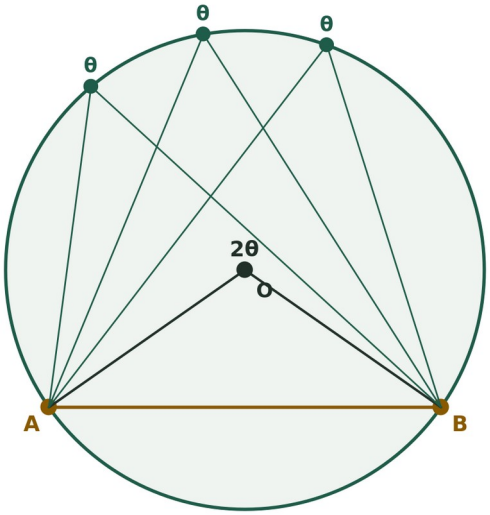
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 25 · The Angle That Does Not Change

Circles

Competencies: C-7.3, CG-9	Suggested time: 75 min	Team of 3–4 · Record everything

Activity 25 · The angle that does not change



Angle at centre = 2 × angle at circumference

Measure six points. Tabulate all six.

Then measure the angle at O.

Compute the ratio each time.

Make AB a diameter

What angle do you get now?

Angle in a semicircle = 90°

— a special case of the rule above.

Cyclic quadrilaterals

Opposite angles are supplementary.

Test it, then use it to check concyclicity.

Every point on the major arc sees AB at the SAME angle θ.

Points on the MINOR arc see AB at a different angle. What is the relationship between the two?

Figure 25 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A photographer wants every seat in a semicircular hall to see the stage at the same angle. A designer says this is impossible. A geometer says it is exactly what a circle does. Your team will test the claim by measurement before proving it.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Large circle drawn on paper or chalked on the ground
>
> Protractor, ruler, string, pins
>
> Coloured pens

EXPLORE AND BUILD

1. Draw a large circle and mark a chord AB. Mark six different points on the major arc.
2. From each point, measure the angle subtended by AB. Record all six.
3. Measure the angle subtended by AB at the centre. Compare with the angles at the circumference.
4. Now mark points on the minor arc and measure the angle subtended there. Compare with the major arc results.

5. Make AB a diameter and repeat. Record the angles obtained.
6. Test whether four chosen points are concyclic by measuring opposite angles of the quadrilateral they form.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Point on arc	Angle subtended by AB (°)	Angle at centre (°)	Ratio	On which arc?

REASON IT OUT

Q1. State the relationship between the angle at the centre and the angle at the circumference, and give your measured evidence.

Q2. Angles in the same segment were equal. Angles in the other segment were not equal to them. State the relationship between the two.

Q3. When AB was a diameter, what angle did you get? State the theorem and explain it as a special case of the centre-angle result.

Q4. State the condition for a quadrilateral to be cyclic, and verify it with your measurements.

PUSH FURTHER

Investigate chords and their distances from the centre: draw five chords of different lengths, measure each chord's length and its perpendicular distance from the centre, and plot one against the other. State the relationship, explain why the diameter is the longest chord, and explain why equal chords are equidistant from the centre.

ROOTS AND REACH

Circles carry meaning as well as geometry — the Dharmachakra, the Ashoka Chakra, and countless architectural motifs. Choose one, examine its geometric construction, and count how many equal divisions it uses. Then work out the angle at the centre for each division.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

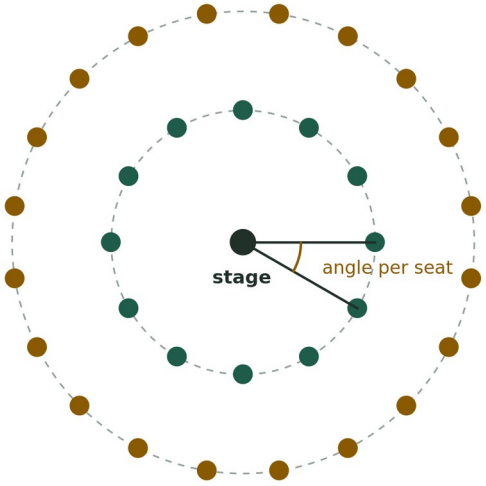
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 26 · The Fair Seating Plan

Circles

Competencies: C-7.3, C-8.3, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 26 · Seating sixty people fairly around a circle



Outer seats sit BETWEEN inner heads
— offset by half a step.

Compute for each ring

- ▸ circumference = 2πr
- ▸ seats = circumference ÷ seat width
- ▸ angle per seat = 360° ÷ seats
- ▸ arc per seat = circumference ÷ seats

Then judge the design

Total must reach 60. Adjust and recompute.

Outer ring is further away — quantify the disadvantage in metres.

Compute the SECTOR area each person gets on each ring, and compare.

Write your method as a numbered algorithm and test it on a different target number.

Figure 26 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A cultural programme must seat 60 people around a circular performance space so that everyone is roughly equidistant from the centre and no one blocks anyone. Your team will design the plan using arcs, chords and angles, and then test it with real people if space allows.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Chalk, long string, measuring tape
Protractor, graph paper, calculator
Chairs or markers if a real trial is possible

EXPLORE AND BUILD

1. Decide a radius for the first ring based on a comfortable viewing distance. Compute the circumference.
2. Given a seat width, compute how many seats fit on the first ring. Compute the angle at the centre per seat.
3. Add a second ring at a larger radius. Compute its circumference and seat count.
4. Design the offset so that second-ring seats look between first-ring heads. Compute the angular offset needed.
5. Compute the arc length between adjacent seats on each ring.

6. Compute the total number of seats and compare with the target of 60. Adjust and recompute.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Ring	Radius (m)	Circumference (m)	Seats	Angle per seat (°)	Arc per seat (m)

REASON IT OUT

Q1. Explain why the ratio of circumference to diameter is the same for both rings, and name that constant.

Q2. Compute the arc length for a 40 degree sector of your outer ring. Show the method.

Q3. Your seat counts probably did not total exactly 60. Describe the adjustment you made and its cost in comfort.

Q4. Someone in the outer ring is further away. Quantify the disadvantage and propose a fix that does not require more space.

PUSH FURTHER

Compute the area of the sector allocated to each person on each ring, and compare. Then compute the area of the segment cut off by a chord joining two adjacent outer seats. Report both, and use them to argue for or against a third ring.

ROOTS AND REACH

Break the seating problem into sub-problems, write your solution as a numbered algorithm, and hand it to another team with a different target number. Check whether their answer is sensible. Report where your algorithm needed judgement that you had not written down.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

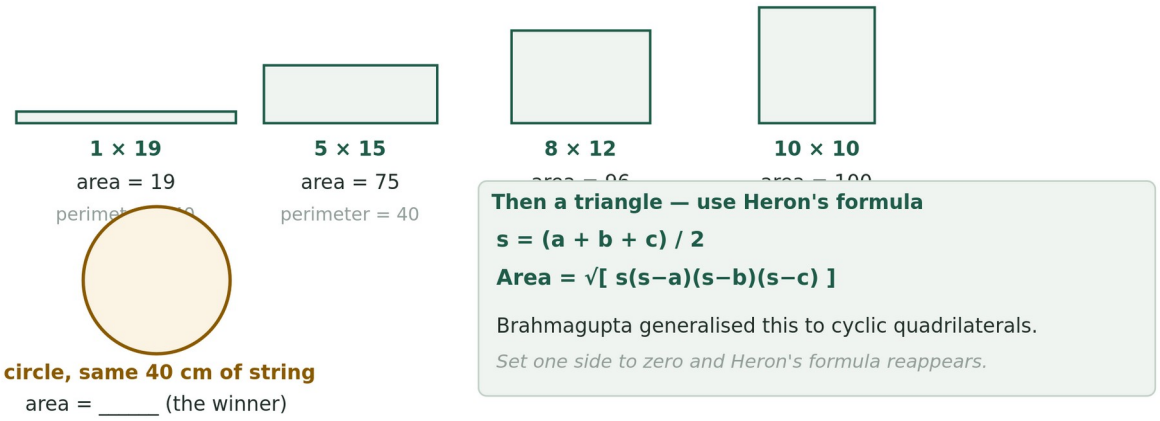
Mensuration: Area and Perimeter

Activity 27 · The Fence and the Field

Mensuration: Area and Perimeter

Competencies: C-5.1, C-8.3, CG-9 | **Suggested time: 75 min** | **Team of 3–4 · Record everything**

Activity 27 · Same fence, different shapes — which encloses most?


1 × 19
area = 19
5 × 15
area = 75
perimeter = 40
8 × 12
10 × 10
circle, same 40 cm of string
area = _____ (the winner)
Then a triangle — use Heron's formula
s = (a + b + c) / 2
Area = √[s(s−a)(s−b)(s−c)]
Brahmagupta generalised this to cyclic quadrilaterals.
Set one side to zero and Heron's formula reappears.

Does shape matter, or only the length of fence? Answer the neighbour with one number.

Figure 27 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A farmer has 40 metres of fencing and wants to enclose the largest possible plot. A neighbour insists that the shape does not matter, only the length of the fence. Your team will test this with string and squared paper, and then explain the result to the neighbour with numbers.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

String cut to exactly 40 cm (representing 40 m), squared paper

Ruler, scissors, calculator

Graph paper

EXPLORE AND BUILD

1. Using the 40 cm loop, form rectangles of dimensions 1x19, 2x18, 5x15, 8x12, 10x10 and others. Check the perimeter is unchanged each time.

2. Compute the area of each rectangle and tabulate.
3. Plot area against one side length. Describe the shape of the curve.
4. Now form the loop into a circle. Compute its radius from the circumference and then its area.
5. Compare the circle's area with the best rectangle's area. Compute the percentage advantage.
6. Try a triangle of perimeter 40 cm and compute its area using Heron's formula.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Shape	Dimensions	Perimeter (cm)	Area (cm^2)	Rank

REASON IT OUT

Q1. Answer the neighbour directly: does shape matter? Give the strongest number from your table.

Q2. Which rectangle gave the maximum area? State the general rule and explain why it is plausible.

Q3. Use Heron's formula to compute the area of a triangle with sides 13, 14 and 15. Show all working.

Q4. Explain why a median of a triangle divides it into two triangles of equal area, using base and height.

PUSH FURTHER

The farmer now has a straight wall he can use as one side, so he only needs to fence three sides. Redo the optimisation with 40 m of fence and one free side. Predict first whether the best shape will change, then compute and check.

ROOTS AND REACH

State Brahmagupta's formula for the area of a cyclic quadrilateral. Then set one side to zero and show that Heron's formula emerges. Explain in your own words what 'special case' and 'generalisation' mean in mathematics, using this as the example.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

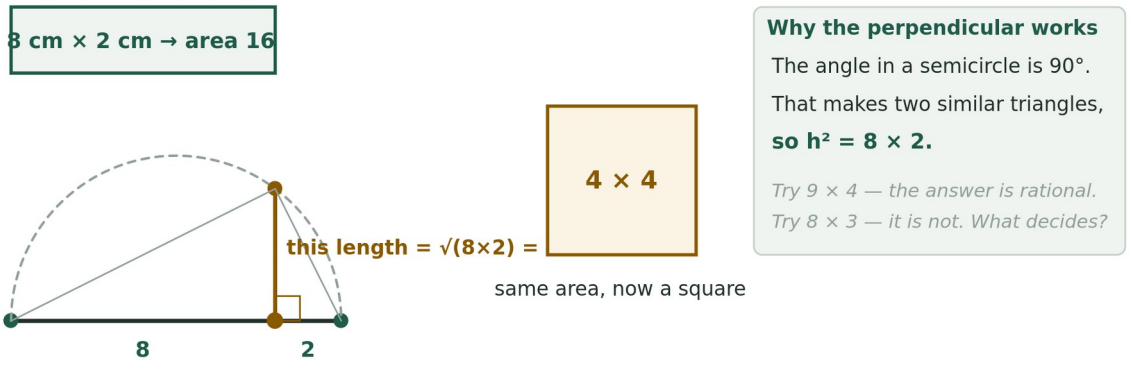
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 28 · Squaring the Rectangle

Mensuration: Area and Perimeter

Competencies: C-5.1, C-7.2	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 28 · Squaring the rectangle (the geometric mean construction)


8 cm × 2 cm → area 16
this length = √(8×2) =
4 × 4
same area, now a square
8
2
Why the perpendicular works
The angle in a semicircle is 90°.
That makes two similar triangles,
so h² = 8 × 2.
Try 9 × 4 — the answer is rational.
Try 8 × 3 — it is not. What decides?

Now try to square a CIRCLE with rope and pegs. Measure how close you get, then find out why.

Figure 28 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

An ancient practical problem: given a rectangular plot, construct a square of exactly the same area, using only rope and pegs. The Sulbasutras solve it. Your team will carry out the construction, verify it by measurement, and then prove it with the identities you built earlier.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Card or paper, scissors, ruler, compass
>
> String, pegs if working on the ground
>
> Calculator

EXPLORE AND BUILD

1. Draw a rectangle of sides 8 cm and 2 cm. Compute its area.
2. Predict the side of the equal-area square. Compute it and note whether it is rational.
3. Carry out the classical construction: place the two sides end to end to make a length of 10 cm, draw a semicircle on it as diameter, and erect a perpendicular at the junction point. Measure where it meets the semicircle.

4. Measure that perpendicular. Compare with your predicted square side.
5. Construct the square on that length and verify its area by cutting and rearranging the original rectangle if possible.
6. Repeat with a rectangle of 9 cm by 4 cm, where the answer is rational, and compare the accuracy.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Rectangle	Area (cm^2)	Predicted square side	Constructed length (cm)	Error	Rational?

REASON IT OUT

Q1. Explain why the perpendicular in a semicircle gives the geometric mean of the two segments. Use the angle-in-a-semicircle theorem.

Q2. Show algebraically that the construction is equivalent to finding the square root of the product of the two sides.

Q3. For the 8 by 2 rectangle the answer was rational. For 8 by 3 it is not. Explain what determines this.

Q4. Explain the classical problem of squaring a shape, and why it mattered practically.

PUSH FURTHER

Attempt to construct a square equal in area to a circle of radius 4 cm using rope and pegs. Get as close as you can, measure your error, and then find out what is known about whether an exact construction is possible. Report what you find.

ROOTS AND REACH

Estimate the area of a circle the way ancient civilisations did: draw a circle on squared paper and count squares, counting partial squares as half. Compare with the formula value, compute your percentage error, and state how many squares per radius you would need for 1% accuracy.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 29 · The Irregular Plot

Mensuration: Area and Perimeter

Competencies: C-5.1, C-8.1, CG-9	Suggested time: 80 min	Team of 3–4 · Record everything

Activity 29 · One irregular plot, three independent methods

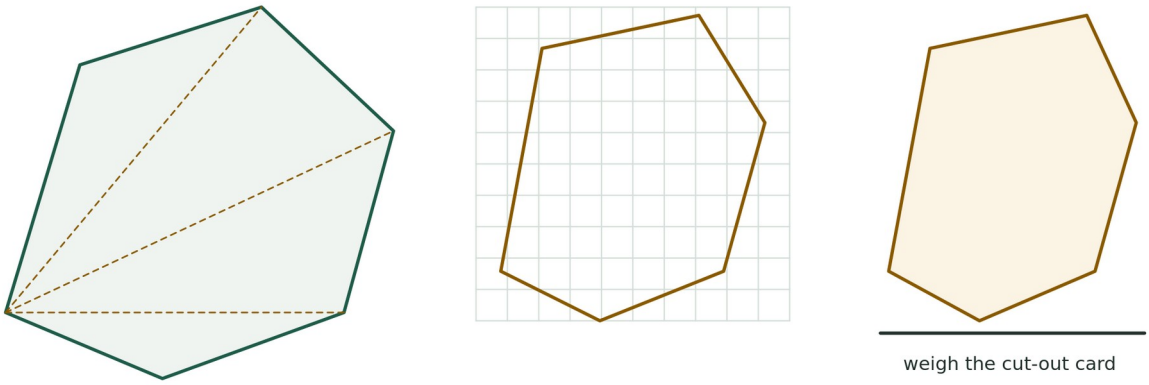

weigh the cut-out card
1 · Triangulate, then Heron on each
2 · Grid counting, with a stated rule for part-squares
3 · Mass method
Three answers will differ. Report: area = _____ ± _____ m² and justify the margin.
A polygon with n sides needs how many triangles? State the general rule.

Figure 29 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A community garden has an irregular boundary and nobody knows its area. Three different people have given three different figures. Your team will measure it properly by at least three independent methods, compare the answers, and state a final figure with an honest uncertainty.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Measuring tape, pegs, string, chalk
>
> Graph paper, tracing paper, ruler
>
> Calculator, weighing balance (for the mass method)

EXPLORE AND BUILD

1. Method 1 — triangulation: divide the plot into triangles by measuring diagonals, and use Heron's formula on each.
2. Method 2 — grid counting: draw the plot to scale on squared paper and count squares, treating partial squares consistently by a stated rule.

3. Method 3 — mass method: cut the scale drawing out of uniform card, weigh it, and weigh a known area of the same card. Scale up.
4. Record all three results in the same units.
5. Compute the mean and the spread of your three values.
6. Identify which method you trust most and state why, before you see whether it agrees with the others.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Method	Measurements used	Area obtained (m^2)	Difference from mean	Main source of error

REASON IT OUT

Q1. Use Heron's formula on one of your triangles and show the full working, including the semi-perimeter.

Q2. Your three answers differed. State your final figure in the form 'value plus or minus uncertainty' and justify the uncertainty.

Q3. Which method would scale best to a plot ten times larger? Which would scale worst?

Q4. Explain why dividing into triangles works for ANY polygon, and state how many triangles a polygon with n sides needs.

PUSH FURTHER

Add a curved boundary: approximate it first by three straight segments, then by six, then by twelve. Record the area each time and plot area against number of segments. Describe what the curve approaches, and connect this to how the area of a circle is derived.

ROOTS AND REACH

Break the whole task into sub-problems and write it as an algorithm someone could follow for any irregular plot. Note carefully at which steps a human has to make a judgement, and consider whether those steps could be made mechanical.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

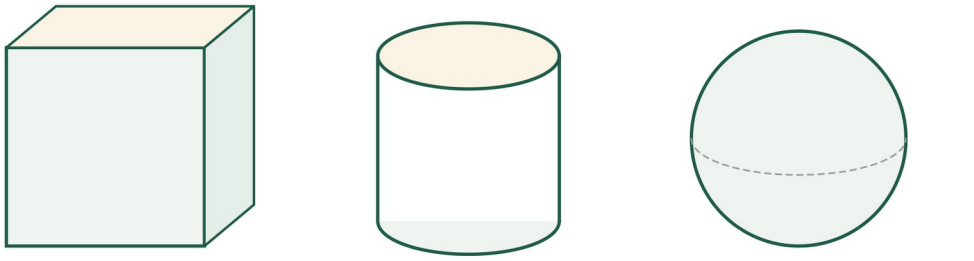
Mensuration: Surface Area and Volume

Activity 30 · The Water Tank Decision

Mensuration: Surface Area and Volume

Competencies: C-5.2, C-8.2, C-8.3 | **Suggested time: 80 min** | **Team of 3–4 · Record everything**

Activity 30 · 1000 litres — which shape wastes least material?



CUBE
surface area = $6a^2$

CYLINDER
$2\pi r^2 + 2\pi rh$

SPHERE
$4\pi r^2$

least material for a given volume

All three hold exactly 1000 litres. Compute the surface area of each and rank them.

The sphere wins mathematically. The market barely sells them. Give three practical reasons.

For the cylinder: plot surface area against radius at constant volume, and read off the minimum.

A cube is a special case of a cuboid. What is the corresponding special case for a cylinder?

Figure 30 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A household must buy a 1000 litre tank. Cylindrical, cuboidal and spherical options are available at different prices. The cost depends on the material used, which depends on surface area. Your team will determine which shape stores 1000 litres using the least material, and then explain why the market does not always sell that shape.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Card, scissors, tape, ruler, compass
Sand or rice, measuring cylinder
Calculator, graph paper

EXPLORE AND BUILD

1. Convert 1000 litres to cubic metres and then to a convenient working unit.
2. For a cube, compute the side needed and then the total surface area.

3. For a cylinder, compute surface area for several different height-to-radius ratios, all holding volume constant. Tabulate.
4. Plot cylinder surface area against radius and find the minimum.
5. For a sphere and a hemisphere, compute the surface area for the same volume.
6. Build scale models of the three shapes and verify their volumes by filling with sand.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Shape	Key dimension(s)	Volume (L)	Surface area (m^2)	Material rank

REASON IT OUT

Q1. Which shape needed least material? State the figures and the percentage saving over the worst option.

Q2. For the cylinder, at what height-to-radius ratio was the surface area minimum? Read it from your graph.

Q3. Explain why the market sells few spherical tanks despite your result. Give at least three practical reasons.

Q4. Explain how a cube is a special case of a cuboid, and identify the corresponding special case relationship for the cylinder.

PUSH FURTHER

Design a combination shape: a cylinder with a hemispherical top. Compute the volume and surface area for a 1000 litre capacity, optimise the ratio, and compare with all your earlier results. State whether the extra manufacturing complexity is justified by the material saved.

ROOTS AND REACH

Use two-dimensional nets to visualise each of your solids. Draw the net of the cylinder and the cone accurately, cut and fold them, and check that the folded surface area matches your calculation. Note any shape whose net you found difficult and explain why.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 31 · Cone, Cylinder, Sphere: The Filling Race

Mensuration: Surface Area and Volume

Competencies: C-5.2, C-8.2	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 31 · The filling race — discover the volume ratios by pouring

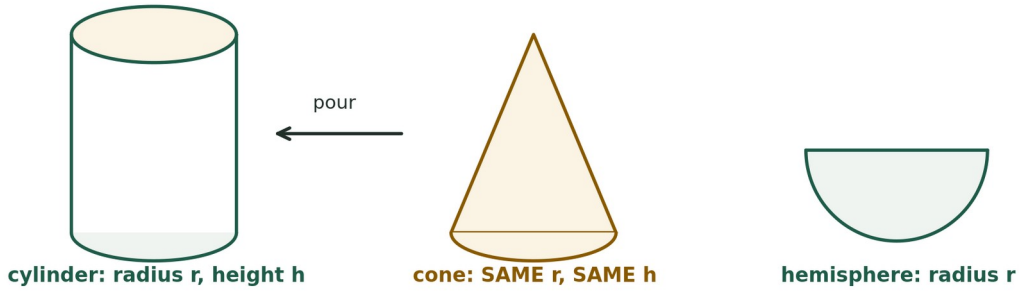

pour
cylinder: radius r, height h
cone: SAME r, SAME h
hemisphere: radius r
How many coneloads fill the cylinder exactly?
So volume of cone = ___ × π r² h
Unroll the cone: you get a SECTOR.
Its arc length equals the base circumference.
Slant height ≠ vertical height. Relate them.
Double r and double h. Predict the volume ratio BEFORE measuring, then test.

Figure 31 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Instead of being given the volume formulae, your team will discover the relationships between them by filling one shape from another. The results are simple ratios that you will then express as formulae and verify.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Card, scissors, tape, compass, ruler
>
> Sand, rice or water; a tray to catch spills
>
> Measuring cylinder

EXPLORE AND BUILD

1. Build a cylinder and a cone with the SAME base radius and the SAME height. Check both carefully.
2. Fill the cone with sand and pour into the cylinder. Count how many coneloads fill the cylinder exactly.
3. Build a hemisphere (or use a ball cut in half) of the same radius. Fill it and pour into the cylinder of the same radius and height equal to the radius. Record the ratio.
4. Express each relationship as a formula and check against the standard formulae.
5. Verify one volume directly by water displacement in a measuring cylinder.

6. Measure the curved surface area of the cone by unrolling it into a sector, and compute its area.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Shape pair	Coneloads / fills	Ratio found	Formula implied	Standard formula

REASON IT OUT

Q1. State the ratio you found between cone and cylinder, and write the cone volume formula from it.

Q2. When you unrolled the cone, you got a sector. What is the arc length of that sector equal to, and why?

Q3. Distinguish clearly between slant height and vertical height, and state the relationship between them.

Q4. Your measured ratio was not exactly the expected value. List three specific causes.

PUSH FURTHER

Investigate scaling: build a second cone with double the radius and double the height. Predict the volume ratio before measuring, then test. Then predict and test the surface area ratio. State the general rule for how area and volume scale with linear dimensions.

ROOTS AND REACH

Apply the scaling rule to a real question: why can a small animal fall a long way unharmed while a large one cannot? Use the area-to-volume ratio in your explanation, and note that this is a mathematical result with biological consequences.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 32 · Packing the Truck

Mensuration: Surface Area and Volume

Competencies: C-5.2, C-8.3, CG-9	Suggested time: 75 min	Team of 3–4 · Record everything

Activity 32 · Packing the truck — square vs hexagonal arrangement

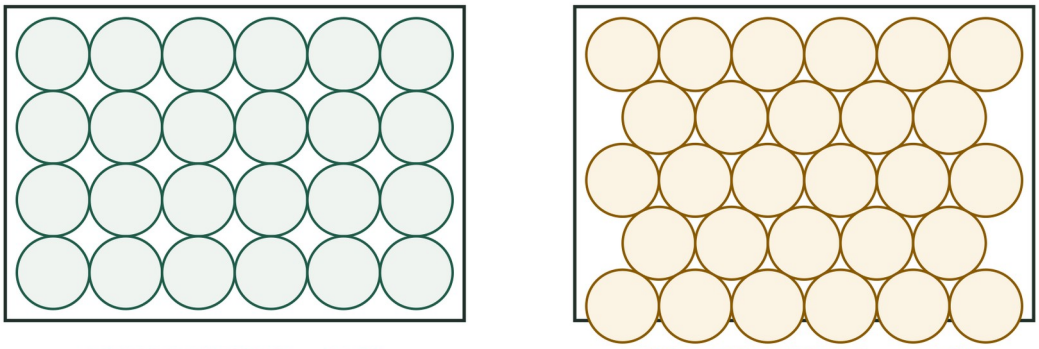

SQUARE PACKING — 24 fit
efficiency = _____ %
HEXAGONAL OFFSET — count it
efficiency = _____ %
efficiency = (total cylinder volume) ÷ (box volume) × 100
Halve the cylinder diameter. Does the COUNT change? Does the EFFICIENCY change? Explain.
Pack the box both ways physically and check your counts before trusting the calculation.

Figure 32 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A supplier must ship cylindrical containers in cuboidal cartons. Currently a lot of air is being shipped. Your team will compute the wasted volume for different packing arrangements and recommend the most efficient — then check whether the recommendation survives a real packing test.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Cylindrical objects: identical bottles, cans or rolled paper tubes
>
> A cuboidal box, ruler, calculator
>
> Graph paper

EXPLORE AND BUILD

1. Measure the box internal dimensions and the cylinder diameter and height. Compute both volumes.
2. Arrangement A — square packing: fit cylinders in rows and columns. Count how many fit and compute total cylinder volume.
3. Arrangement B — hexagonal offset packing: offset alternate rows. Count how many fit.

4. For each arrangement compute the packing efficiency as cylinder volume divided by box volume, as a percentage.
5. Physically pack the box both ways and check your counts.
6. Repeat the calculation for cylinders of half the diameter and see whether efficiency changes.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Arrangement	Cylinders fitted	Cylinder volume total	Box volume	Efficiency %	Wasted volume

REASON IT OUT

Q1. Which arrangement was more efficient, and by how much? Did the physical test agree with the calculation?

Q2. Halving the cylinder diameter changed the count. Did it change the EFFICIENCY? Explain the result.

Q3. Compute the maximum possible efficiency for packing circles in a plane and compare with what you achieved.

Q4. The supplier pays by volume shipped. Convert your efficiency gain into a percentage cost saving and state any assumption.

PUSH FURTHER

Extend to a real constraint: the cartons must also be stackable and must not be crushed. Investigate whether the more efficient arrangement is also the stronger one by loading both packed boxes until deformation. Report the trade-off.

ROOTS AND REACH

Systematically vary one dimension at a time and record the effect on efficiency. Present your findings as a table and a short rule of thumb the supplier could apply without doing any calculation.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

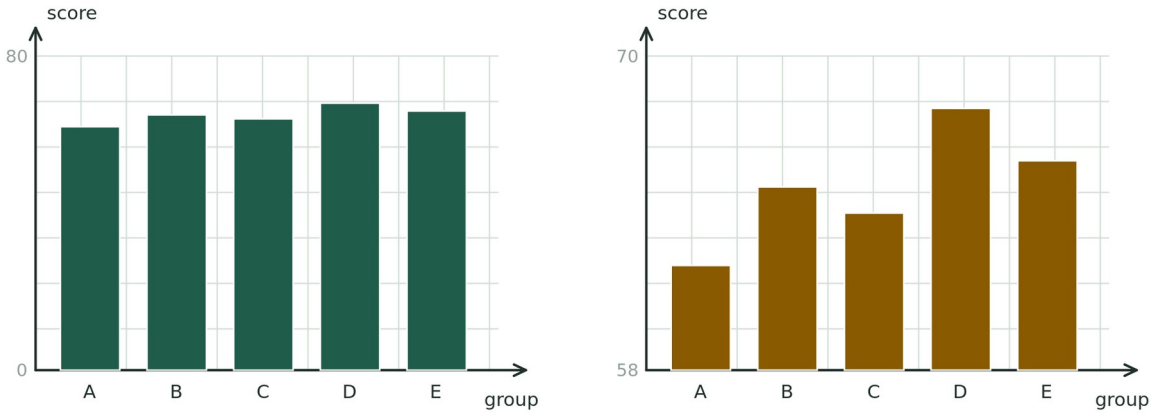
Statistics

Activity 33 · The Survey That Asks the Wrong Question

Statistics

Competencies: C-6.1, CG-9	Suggested time: 2 sessions	Team of 3–4 · Record everything

Activity 33 · The same data, two very different graphs



axis starts at ZERO
'barely any difference'

axis starts near the minimum
'a dramatic gap!'

Mean · Median · Mode — compute all three, then add ONE extreme value and recompute. Which moves

Also run two versions of your survey question — one neutral, one leading — and compare the answers.

Figure 33 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

Two teams survey the same students about canteen food and get opposite conclusions. The difference is in the questions. Your team will design a survey properly, collect real data, compute the measures of central tendency, and demonstrate how the same data can be presented to support two different stories.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Survey slips, clipboard

Graph paper, coloured pens, calculator

Spreadsheet if available

EXPLORE AND BUILD

1. Write two versions of the same question — one neutral, one leading. Run both on different groups of 20 students.

2. Compare the response distributions. Record the difference caused by wording alone.
3. Using the neutral version, collect a numerical variable from at least 40 students, such as minutes spent travelling to school.
4. Compute the mean, median and mode. Show the working for each.
5. Draw a bar graph, then a stacked bar graph showing the same data split by another category.
6. Now draw the same data twice: once with the vertical axis starting at zero, once starting near the minimum value.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Value / class interval	Frequency	Cumulative frequency	Contribution to mean

REASON IT OUT

Q1. Which measure of central tendency best represents your data, and why? Consider whether there are extreme values.

Q2. Add one extreme value to your data set and recompute all three measures. Which changed most? Explain.

Q3. Your two graphs of the same data look very different. Explain exactly what changing the axis does to the reader's impression.

Q4. Compute a weighted average for a situation where the groups are of unequal size, and explain why a simple average would mislead.

PUSH FURTHER

Formulate a genuine statistical investigative question about your school, collect the data, choose the representation yourself, and present the finding to a real audience. Include one sentence stating what your data does NOT show.

ROOTS AND REACH

Find a graph in a newspaper or on a public dashboard. Identify the axis range, the sample size and the source. Redraw it honestly if you find a distortion, and write two sentences on what changed.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

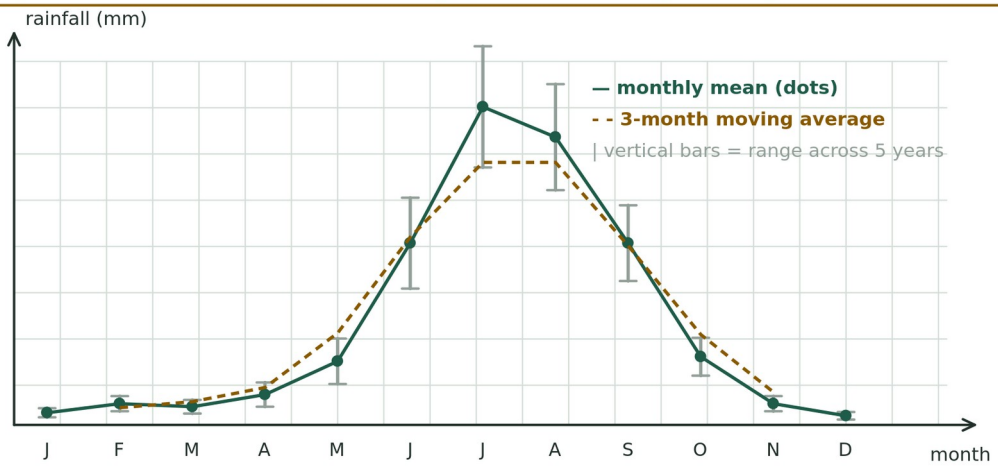
Teacher's initials _____ Date _____ Self-rating (1–5): ____

Activity 34 · Reading a Real Data Set

Statistics

Competencies: C-6.1, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 34 · Monthly means, the range, and a moving average



The wettest month and the most VARIABLE month are usually not the same one.

Advise the farmer on the basis of the range, not the mean. State one warning.

A moving average smooths the line by throwing information away. Say what it discards.

Figure 34 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A district officer has rainfall figures for twelve months across five years and must advise farmers on sowing dates. She cannot read the table. Your team will turn it into something usable and make an actual recommendation, with the uncertainty stated.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> A real data table: rainfall, temperature, or school attendance over time
>
> Graph paper, ruler, coloured pens
>
> Calculator

EXPLORE AND BUILD

1. Organise the raw data into a clear table with consistent units.
2. Compute the mean for each month across the years, and the range.
3. Plot monthly means as a line graph, and add the range as vertical bars at each point.
4. Identify the month with the highest mean and the month with the greatest variability. Note that these are usually not the same.

5. Compute a 3-month moving average and plot it alongside the raw means.
6. Draw a 100% stacked bar chart showing the share of annual total falling in each season.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Month	Year 1	Year 2	Year 3	Year 4	Year 5	Mean	Range

REASON IT OUT

Q1. Which month is most reliable for planning, and which is least? Base your answer on the range, not the mean.

Q2. Explain what a moving average does to the data and what information it deliberately discards.

Q3. Interpret your 100% stacked bar chart in one sentence. What does it show that the line graph does not?

Q4. Give the officer one clear recommendation and one clear warning, each supported by a specific figure.

PUSH FURTHER

Extend the data set with a sixth year if available, or hold back one year from your analysis, make a prediction from the first four, and then check against the fifth. Report your prediction error honestly and comment on whether five years is enough data.

ROOTS AND REACH

Apply computational thinking: write down the exact sequence of steps you used to go from raw table to recommendation. Give it to another team with a different data set and see whether they reach a defensible conclusion by following it.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Introduction to Probability

Activity 35 · Is This Coin Fair?

Introduction to Probability

Competencies: C-6.2, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

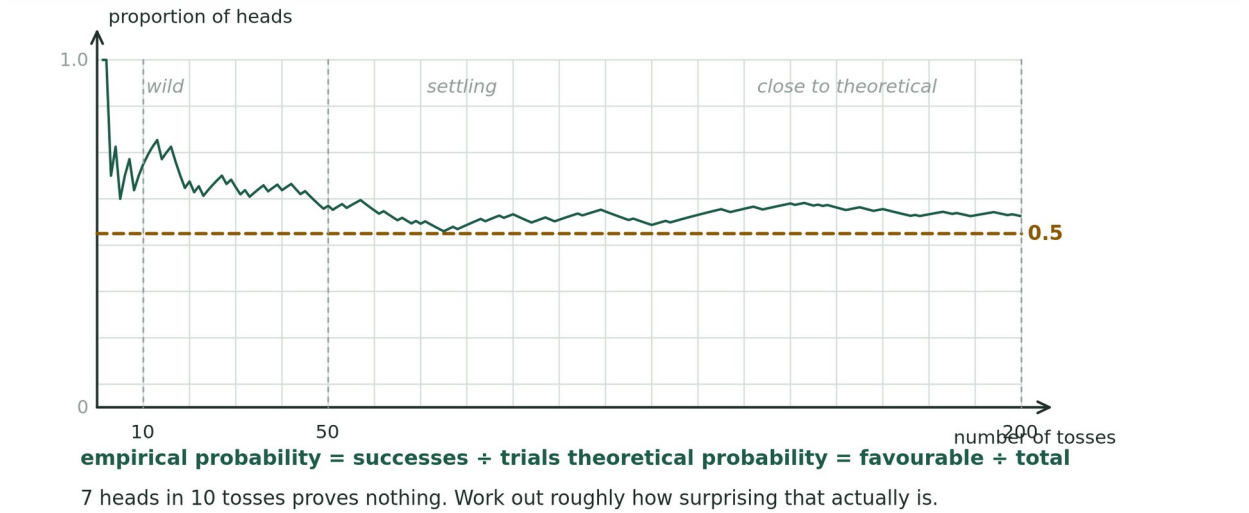

Activity 35 · Is this coin fair? Watch the proportion settle
proportion of heads
1.0
0
wild
settling
close to theoretical
0.5
10
50
200
number of tosses
empirical probability = successes ÷ trials theoretical probability = favourable ÷ total
7 heads in 10 tosses proves nothing. Work out roughly how surprising that actually is.
Decide in advance how many tosses and what proportion would convince you. Then test blind.

Figure 35 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A student claims his coin lands heads more often. Everyone has an opinion; nobody has data. Your team will settle it — and in the process discover why a small number of trials proves nothing and how many you actually need.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Coins, dice, a bag with coloured counters
>
> Tally sheets, calculator, graph paper

EXPLORE AND BUILD

1. Toss one coin 10 times. Record the proportion of heads. Then 50 times. Then 200 times, pooling across the class if needed.
2. Plot the running proportion of heads against the number of tosses. Watch the curve settle.
3. Compute the theoretical probability and mark it on the same graph.

4. Repeat with a die: record the frequency of each face over 120 rolls and compare with expectation.
5. Now a bag with an unknown mixture of counters: draw with replacement 60 times and estimate the composition. Then open the bag and check.
6. Record how far your estimate was from the truth.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Trials	Heads	Proportion	Distance from 0.5	Running estimate

REASON IT OUT

Q1. At what number of trials did your running proportion stop swinging wildly? State the number and describe the shape of the curve.

Q2. Distinguish empirical probability from theoretical probability, using your coin and die results.

Q3. In the bag experiment, how accurate was your estimate? What would have made it better, and what would that have cost?

Q4. Someone tosses 7 heads in 10 tosses. Is the coin unfair? Compute how surprising that actually is, at least roughly.

PUSH FURTHER

Design a test to detect a genuinely biased coin. Decide in advance how many tosses you will do and what proportion would convince you. Then have a partner secretly weight one coin, run your test blind, and see whether your rule detects it.

ROOTS AND REACH

Randomness is used to make decisions fair — in lotteries, in sampling, in queuing. Find one real use, describe how randomness is generated there, and consider what would go wrong if it were not truly random.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

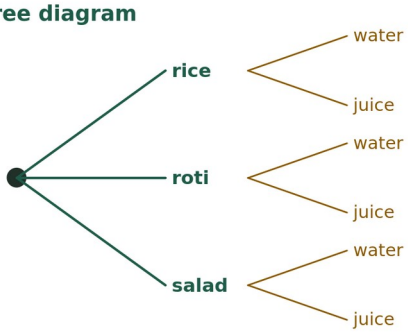
Activity 36 · Trees, Tables and Two Events

Introduction to Probability

Competencies: C-6.2, CG-9	Suggested time: 70 min	Team of 3–4 · Record everything

Activity 36 · One sample space, three representations

1 · Tree diagram



3 branches × 2 branches = 6 outcomes

Add a third stage: how many branches now?

2 · Table

	water	juice
rice		
roti		
salad		

3 · Or simply list all six outcomes.

Which would you choose for THREE events?

Then redo it drawing WITHOUT replacement.

'At least one' is usually easier via the complement. Try it both ways and compare.

Figure 36 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

A canteen offers a meal with a random side dish and a random drink. A student wants to know the chance of getting his least favourite combination. Your team will build the sample space three ways — by listing, by table, and by tree diagram — and decide which method is best for which situation.

BEFORE YOU BEGIN — COMMIT TO A PREDICTION

Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

GET YOUR HANDS ON IT

Cards or slips representing outcomes, two bags

Paper, coloured pens, ruler

Calculator

EXPLORE AND BUILD

1. Set up two random events with 3 and 4 outcomes respectively. List every possible combination.
2. Represent the sample space as a table with rows and columns.
3. Represent the same sample space as a tree diagram.
4. Compute the probability of several events: a specific combination, at least one of a kind, and none of a kind.
5. Now run the experiment 60 times with actual draws and record the observed frequencies.

6. Compare observed frequencies with theoretical probabilities and compute the difference for each outcome.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Outcome	Theoretical probability	Expected in 60 trials	Observed	Difference

REASON IT OUT

Q1. Which representation — list, table or tree — was clearest for this problem? Which would you choose for three events instead of two?

Q2. Compute the probability of 'at least one' event and explain why computing the complement is often easier.

Q3. Your observed and expected counts differed. Is that evidence that the draws were not random? Explain carefully.

Q4. Extend the tree to a third stage. How many branches are there now, and what is the general rule?

PUSH FURTHER

Change the experiment to drawing without replacement and rebuild the tree. Compute the probabilities again and state precisely how and why they changed. Then describe a real situation where each version applies.

ROOTS AND REACH

Use simulation to check a probability that would be tedious to compute: choose a question, simulate 100 trials by hand or with a spreadsheet, and compare the simulated answer with the exact one. Report which was faster and which you trust more.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Integrated Project

Activity 37 · Redesigning a Real Space

Integrated Project

Competencies: C-5.1, C-5.2, C-6.1, C-8.1, C-8.3, CG-9 | **Suggested time: 3 weeks** | **Team of 3–4 · Record everything**

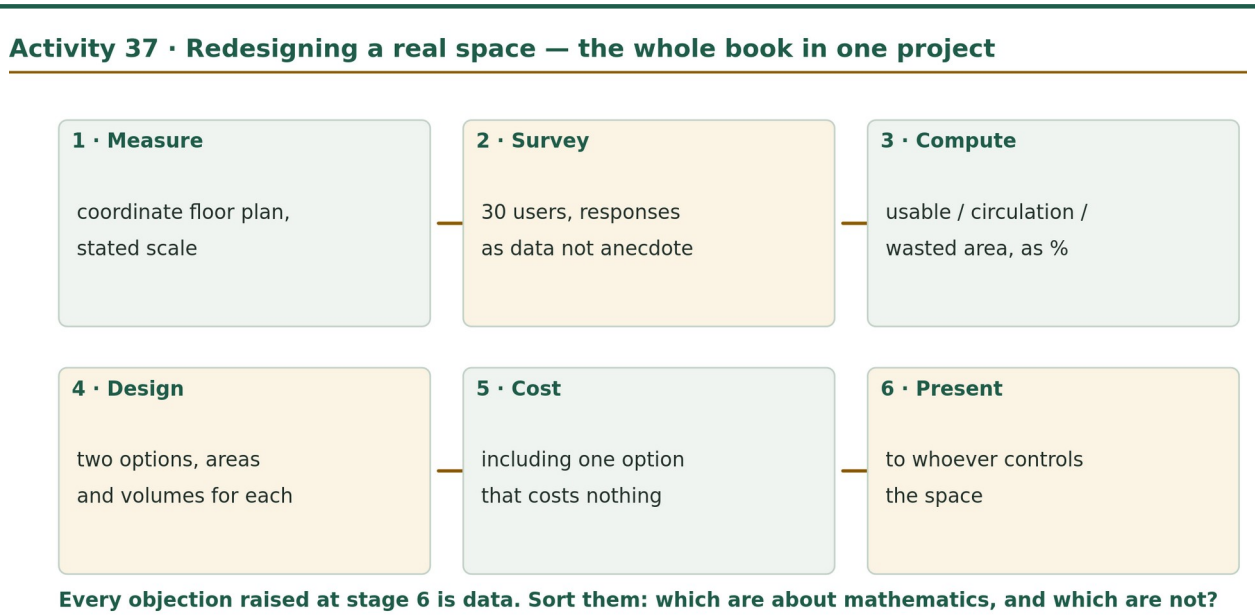

Activity 37 · Redesigning a real space — the whole book in one project
1 · Measure
coordinate floor plan, stated scale
2 · Survey
30 users, responses as data not anecdote
3 · Compute
usable / circulation / wasted area, as %
4 · Design
two options, areas and volumes for each
5 · Cost
including one option that costs nothing
6 · Present
to whoever controls the space
Every objection raised at stage 6 is data. Sort them: which are about mathematics, and which are not?
Identify the assumption in your costing most likely to be wrong, and estimate how wrong.

Figure 37 — study this before you read the steps. It is a map of the activity, not the answer.

THE SITUATION

This closing project uses the whole book. Choose one real space — a classroom, a canteen, a garden, a library corner, a bus stop near your school — and redesign it. You will measure it, model it with coordinates, compute areas and volumes, survey its users, cost your proposal, and present it to whoever actually controls that space.

> **BEFORE YOU BEGIN — COMMIT TO A PREDICTION**
>
> Write what you expect to happen and why. Do not change this later. Being wrong here is not a penalty; it is the most useful line on the page.

> **GET YOUR HANDS ON IT**
>
> Measuring tape, graph paper, camera
> Survey slips, calculator, spreadsheet if available
> Card and materials for a scale model

EXPLORE AND BUILD

1. Measure the space fully and produce a coordinate floor plan to a stated scale.

2. Survey at least 30 users about what does not work. Record responses as data, not anecdotes.
3. Compute current usable area, circulation area and wasted area. Express each as a percentage.
4. Produce two competing redesigns. For each, compute the new usable area and any material or volume required.
5. Cost both options. Include a no-cost option that only rearranges what exists.
6. Build a scale model of your preferred option and compute its volume of any new construction.

RECORD AND COMPUTE

Fill this in as you work — not afterwards from memory.

Measure	Current	Option A	Option B	Change	Cost (Rs)

REASON IT OUT

Q1. Which single measurement best captures the current problem? Justify choosing it over the alternatives.

Q2. Present your survey results using an appropriate measure of central tendency and explain the choice.

Q3. Your two options involve a trade-off. State it precisely, with numbers on both sides.

Q4. Identify the assumption in your costing that is most likely to be wrong, and estimate how wrong.

PUSH FURTHER

Present to the person who controls the space. Record every objection raised. Then revise the proposal to address at least two objections, recompute the affected figures, and note which objections were about mathematics and which were about something else entirely.

ROOTS AND REACH

Look back across all thirty-seven activities. You have measured before assuming, predicted before testing, built before proving, and stated uncertainty rather than hiding it. Write half a page naming the two habits you will carry forward, with a specific example of each from your own work in this book.

Working Space, Sketches and Observations

Use this page for your diagram, your graph, your rough working, and anything that surprised you. Surprises are data.

Diagram / graph / model sketch

What did not go as planned, and what did you do about it?

One question this activity left you with:

Teacher's initials _____ Date _____ Self-rating (1–5): ____

Appendix: Assessment and Record Sheets

Use this sheet across the year. The third column matters most: estimating or predicting before computing is what turns an exercise into an investigation.

Activity no.	Date	Prediction made before testing? Y/N	Data recorded fully? Y/N	Reasoning quality 1–5	Teacher remark

Portfolio Reflection

Name one activity where the physical result and the algebraic result agreed, and explain why that agreement is not a coincidence.

Describe one occasion in this book where you believed something because a diagram looked right, and were later shown a stronger reason.

Which proof in this book did you find genuinely convincing, and what made it convincing rather than merely correct?

Where did a real measurement refuse to match your model? What did you leave out of the model?

Name one mathematical idea in this book that you first met as something you built with your hands. Does that change how well you remember it?